EECS498-003
Formal Verification of Systems Software

Material and slides created by

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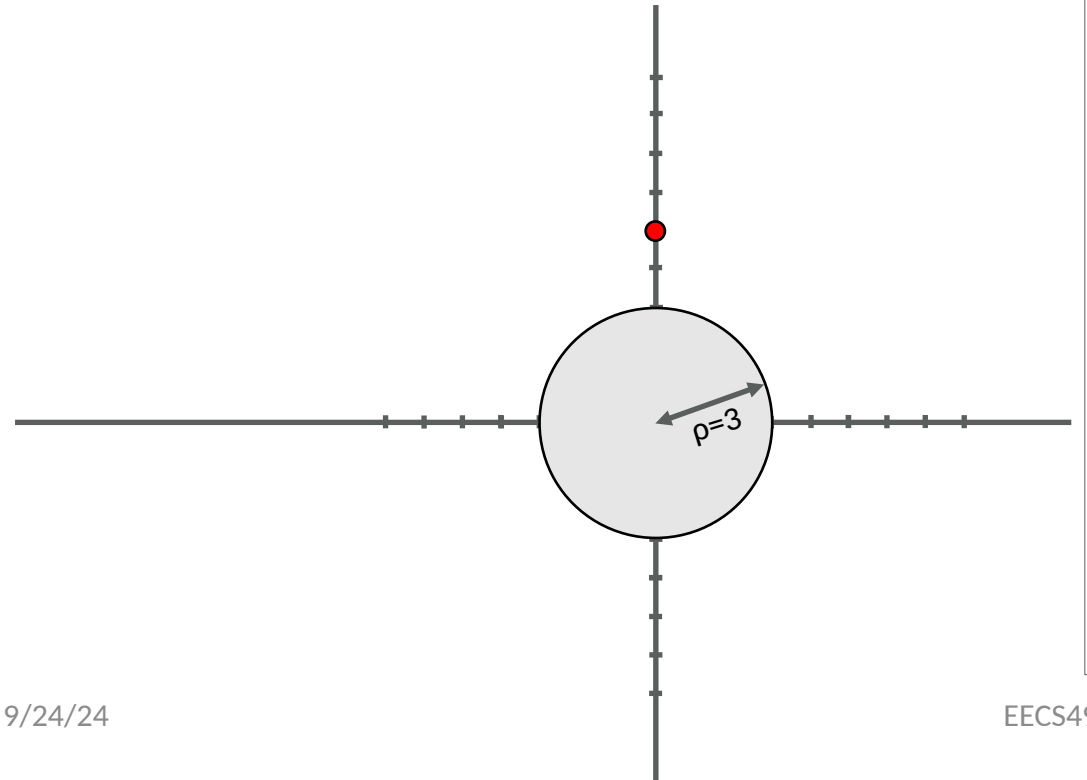
Chapter 4

Inductive Invariants

A simple application: Crawler

- Crawler starts at (0,5)
- It can move 1 step north or 1 step south-east
- Can it ever fall in the hole?

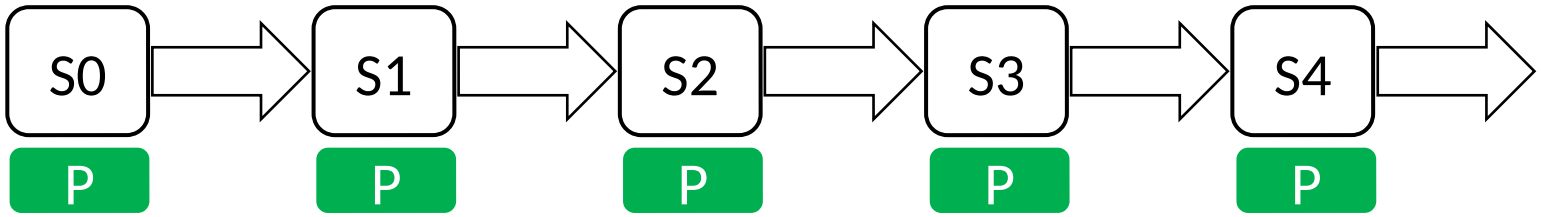



```
datatype Variables = Variables(x:int, y:int)
predicate Init(v:Variables) {
  && v.x == 0
  && v.y == 5
}

predicate MoveNorth(v:Variables, v':Variables)

{
  && v'.x == v.x
  && v'.y == v.y + 1
}

predicate MoveSouthEast(v:Variables,

v':Variables) {
  && v'.x == v.x + 1
  && v'.y == v.y - 1
}
```

Proving invariants

Proof by induction

- Prove it holds on the first state
- Prove it holds during a transition



Init(v) ==> P(v)

P(v) && Next(v, v') ==> P(v')

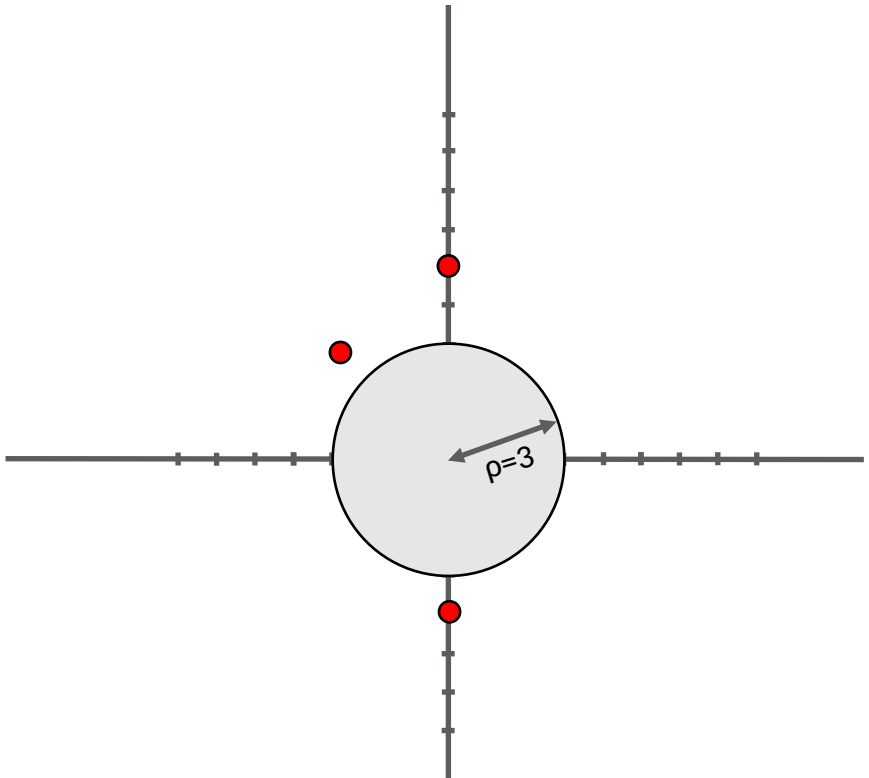
Proving the Crawler

```
datatype Variables = Variables(x:int, y:int)
predicate Init(v:Variables) {
  && v.x == 0
  && v.y == 5
}

predicate MoveNorth(v:Variables, v':Variables)

{
  && v'.x == v.x
  && v'.y == v.y + 1
}

predicate MoveSouthEast(v:Variables,

v':Variables) {
  && v'.x == v.x + 1
  && v'.y == v.y - 1
}
```

```
predicate Safety(v:Variables) {
}    v.x*v.x + v.y*v.y > 3*3
}
```

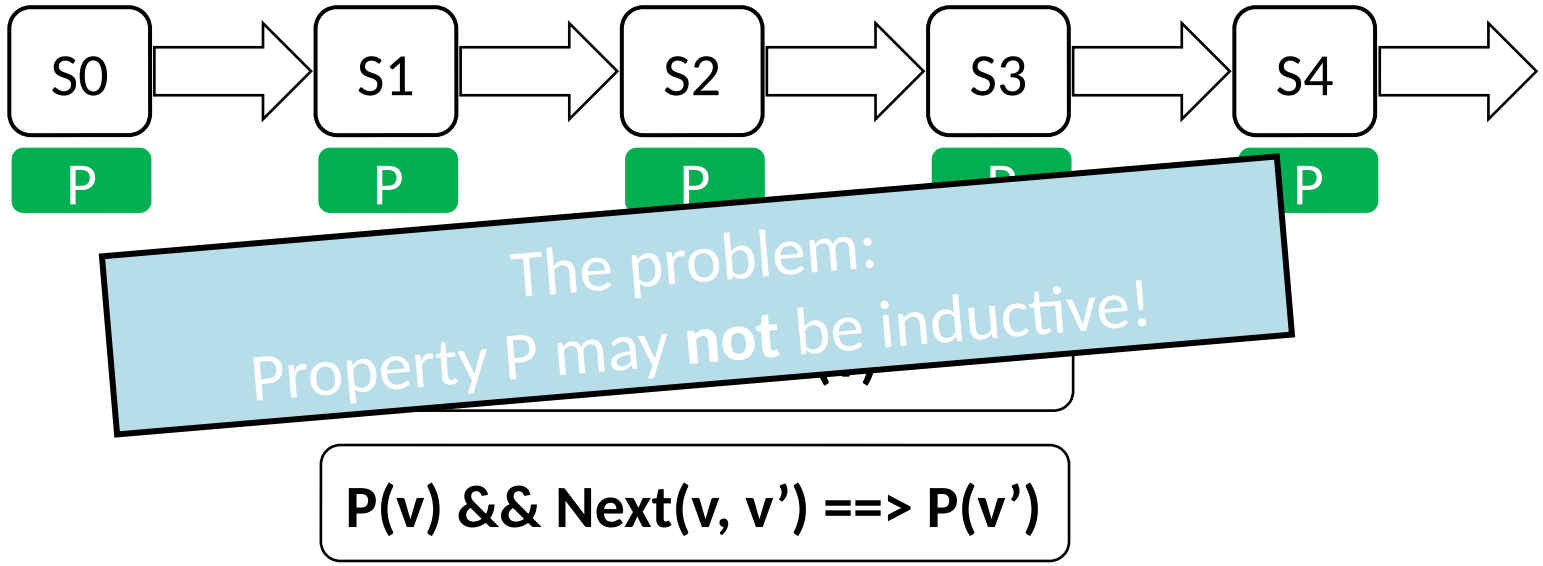
Init(v) ==> P(v) ✓

P(v) && Next(v, v') ==> P(v') ✗



Inductive invariants

Safety property (a.k.a. invariant):
a property that **always** holds

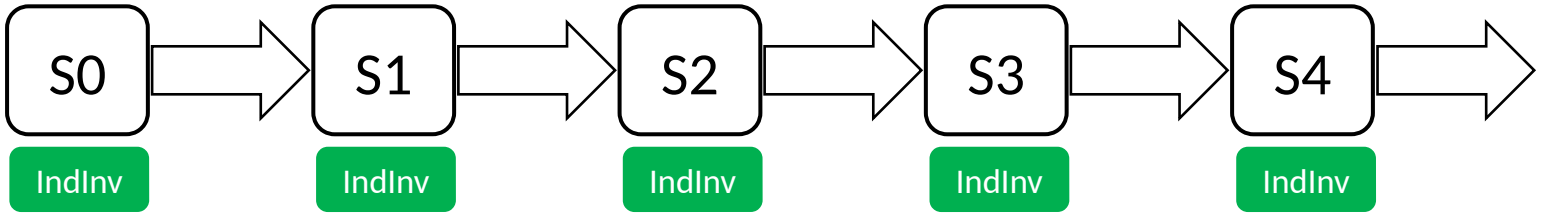


Proving safety with inductive invariants

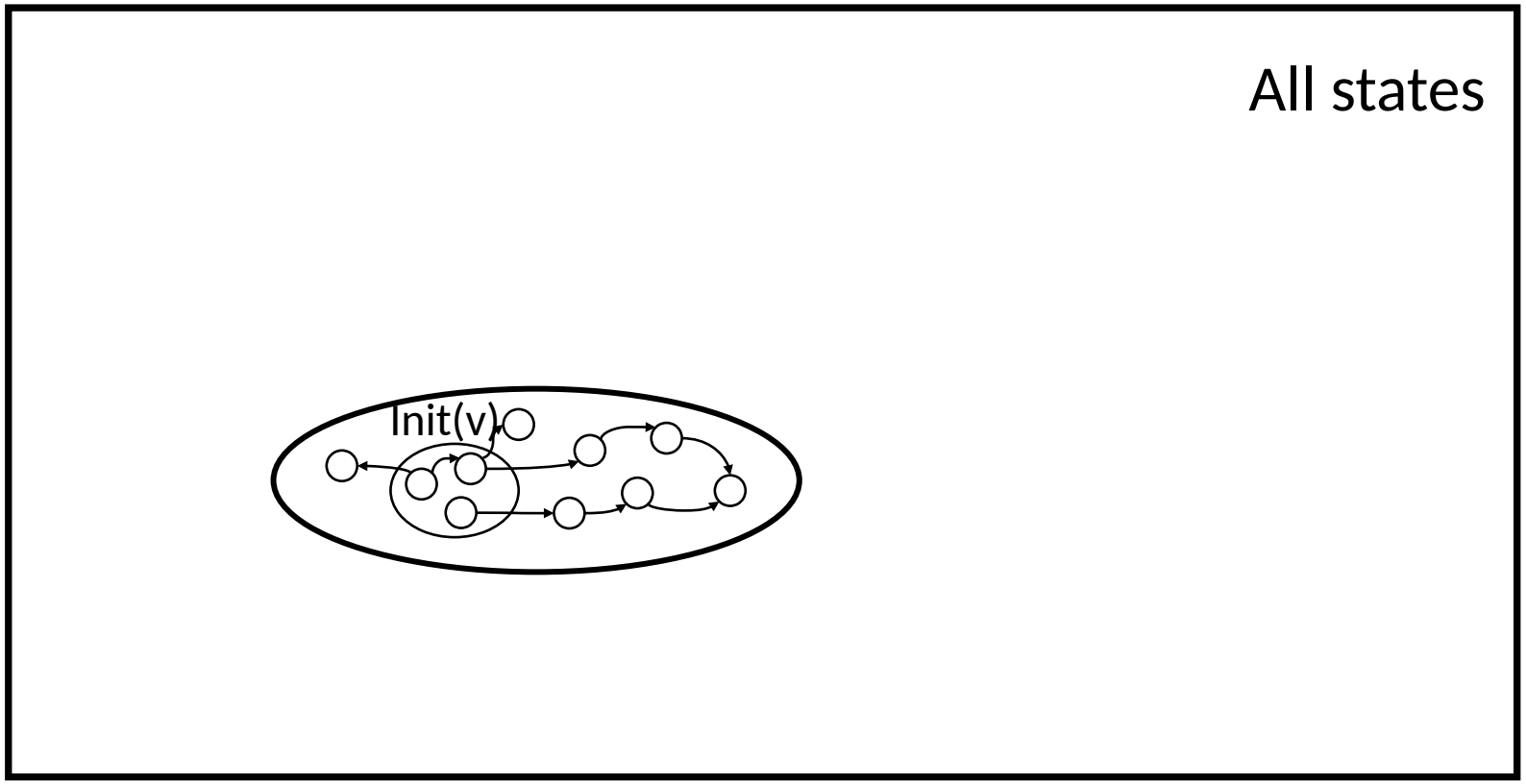
IndInv(v) ==> Safety(v)

Init(v) ==> IndInv(v)

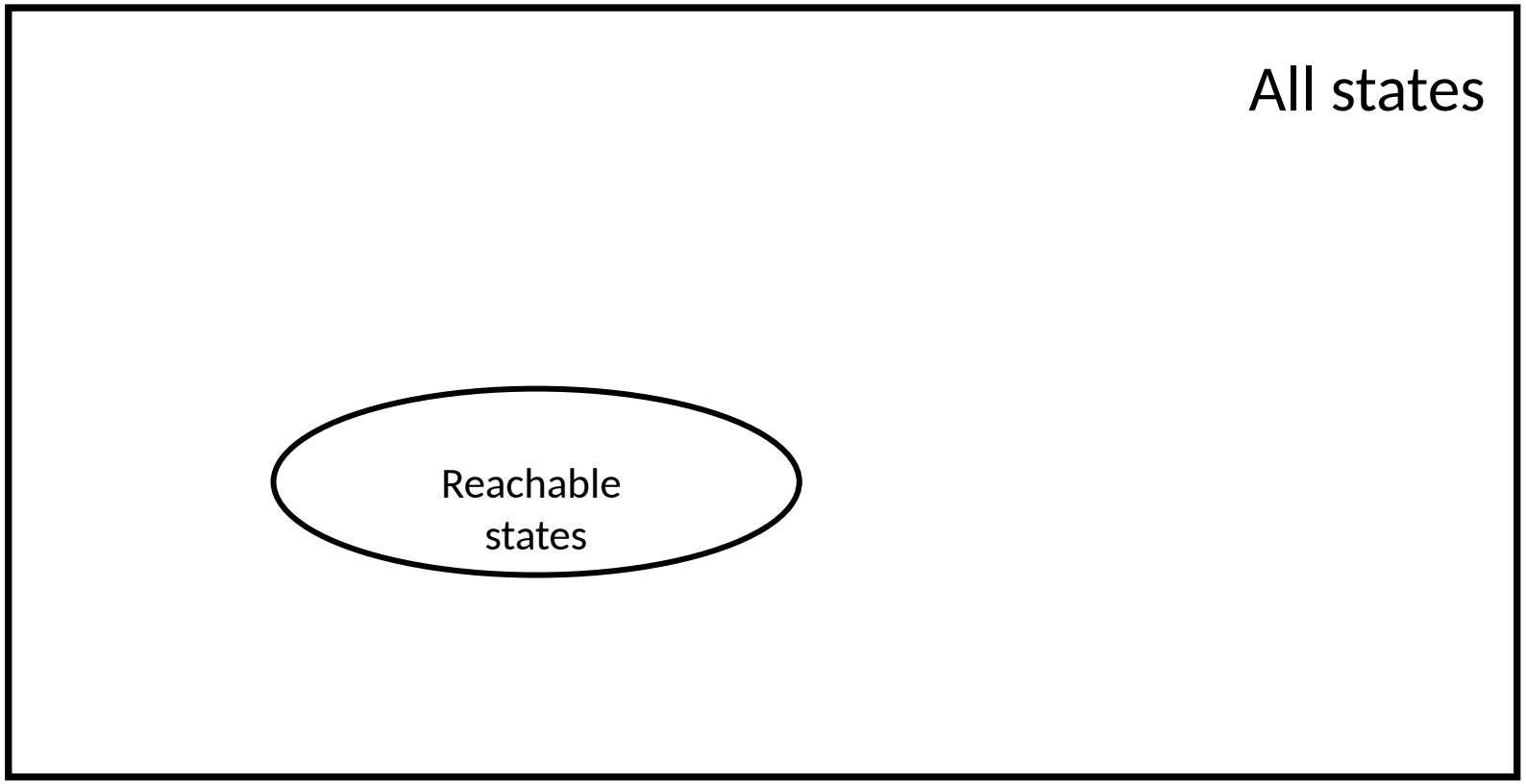
IndInv(v) && Next(v, v') ==> IndInv(v')



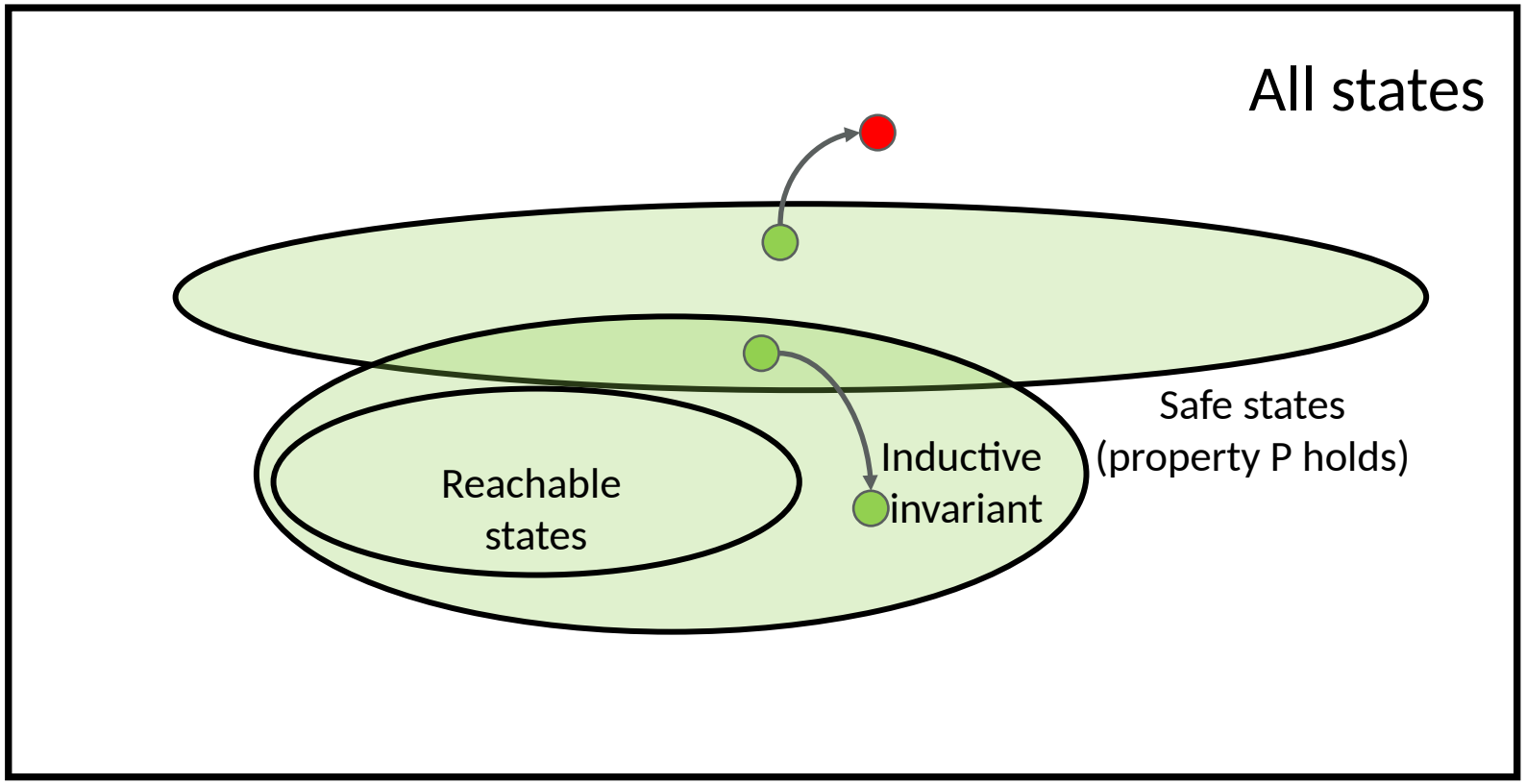
Invariants vs Inductive invariants



Invariants vs Inductive invariants

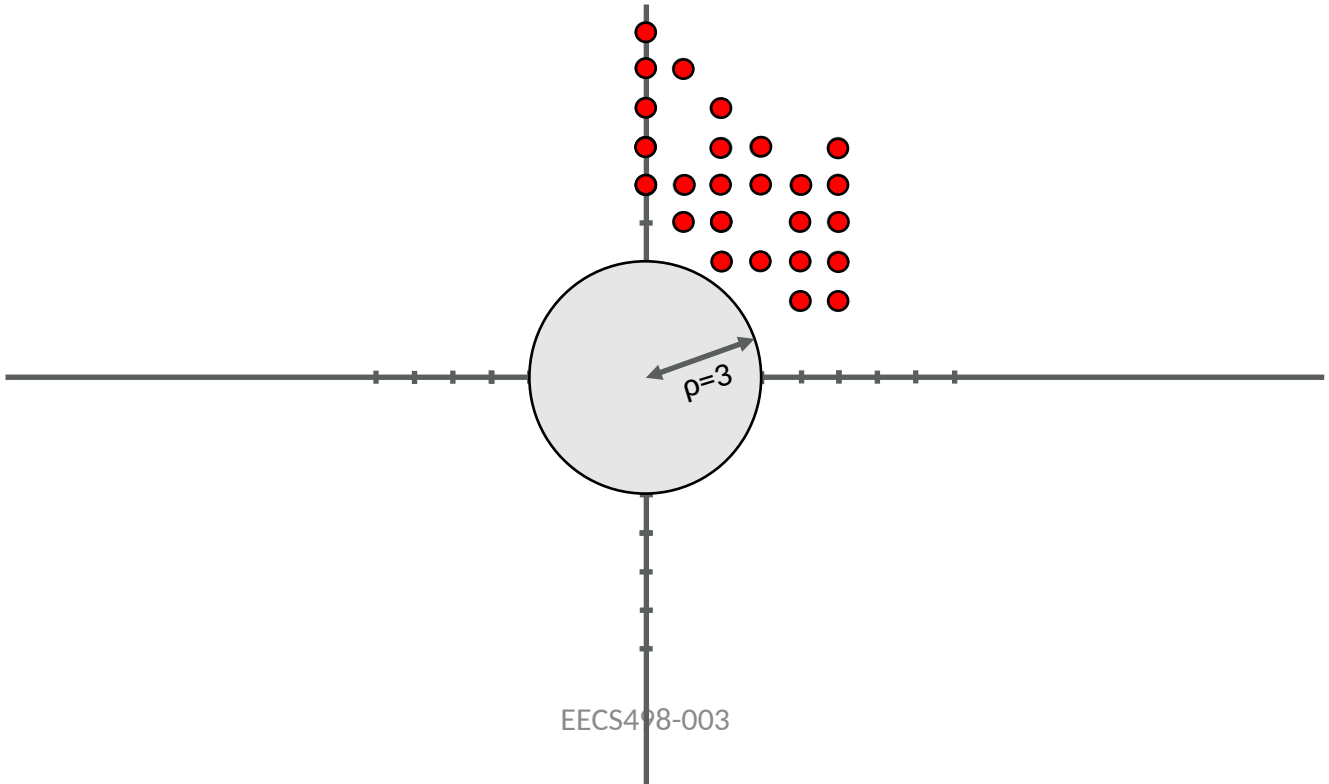


Invariants vs Inductive invariants



Proving the Crawler

Can the crawler ever fall in the hole?



Proving the Crawler

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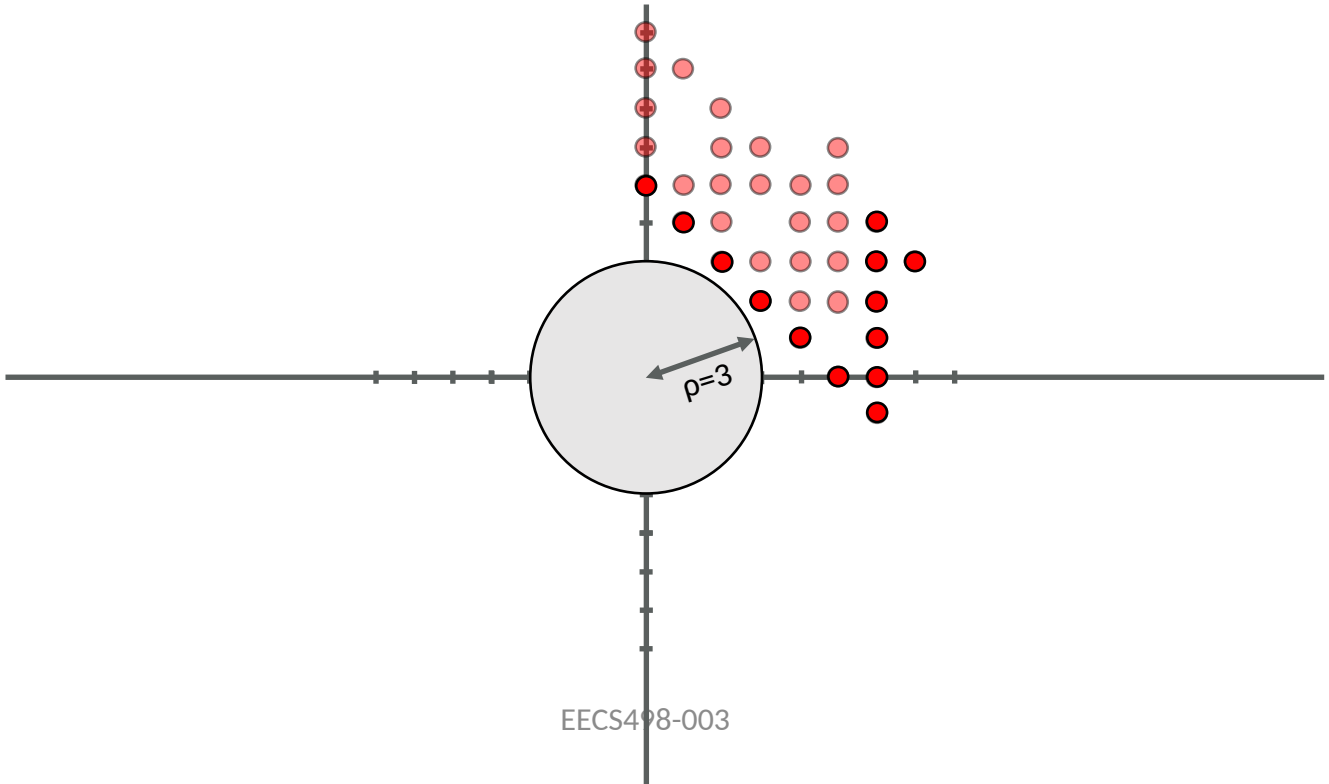
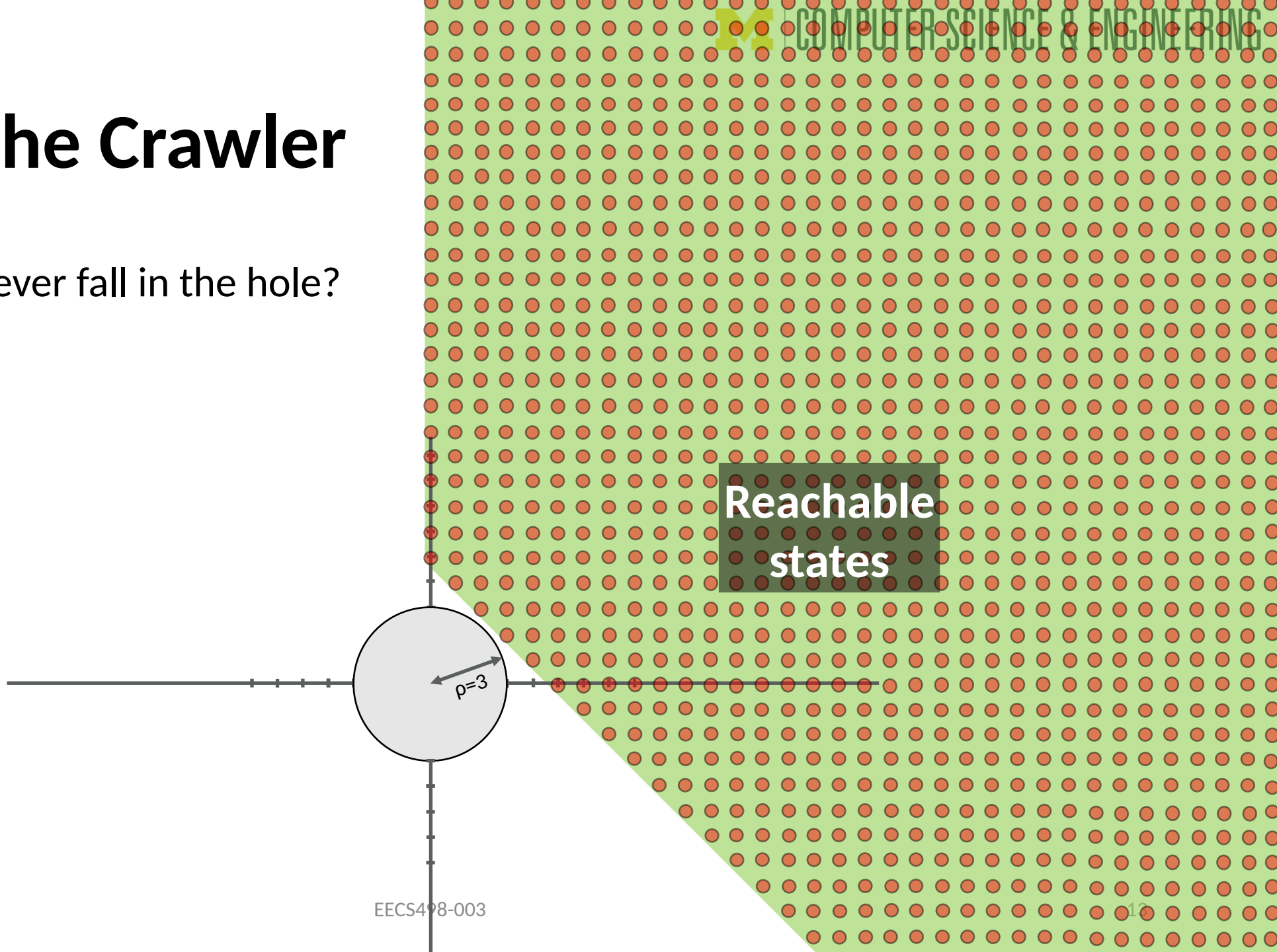


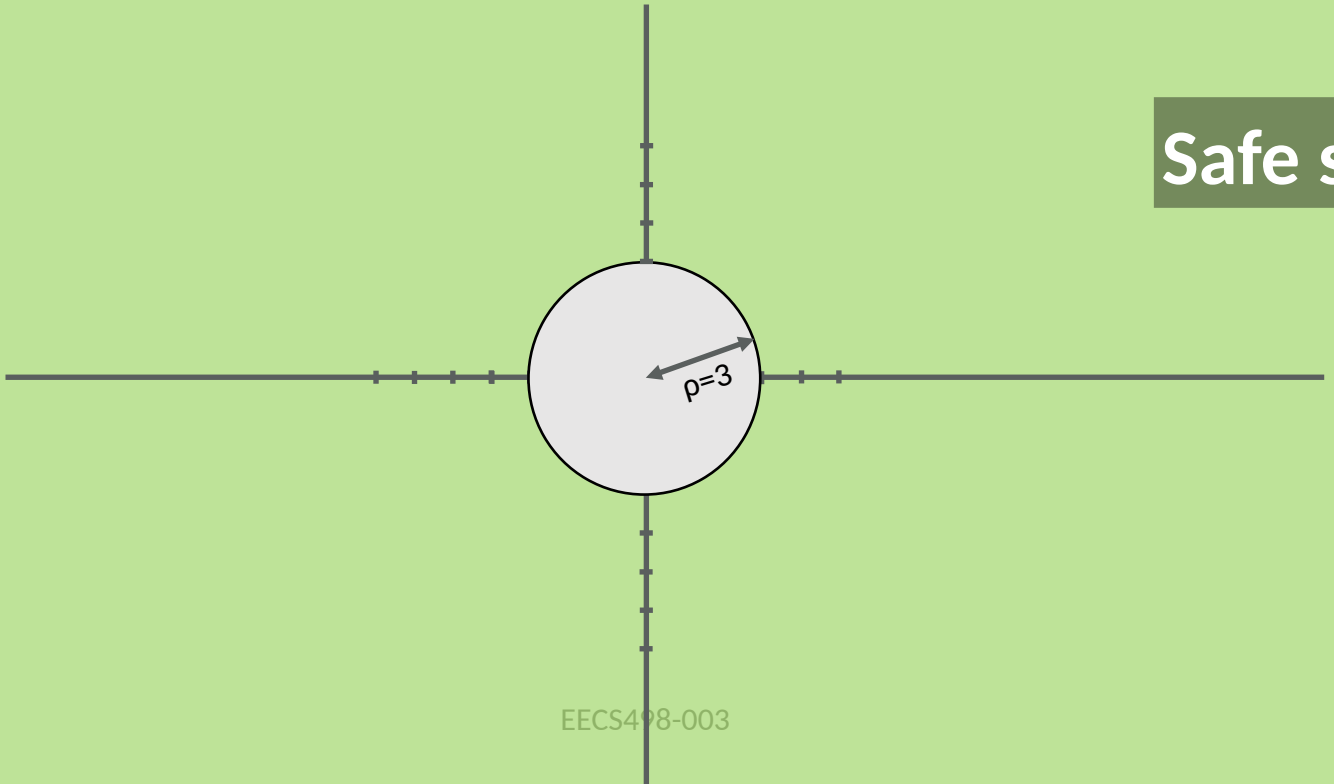
Proving the Crawler

Can the crawler ever fall in the hole?



Naïve safety proof

Safe(v) && Next(v, v') ==>
Safe(v')

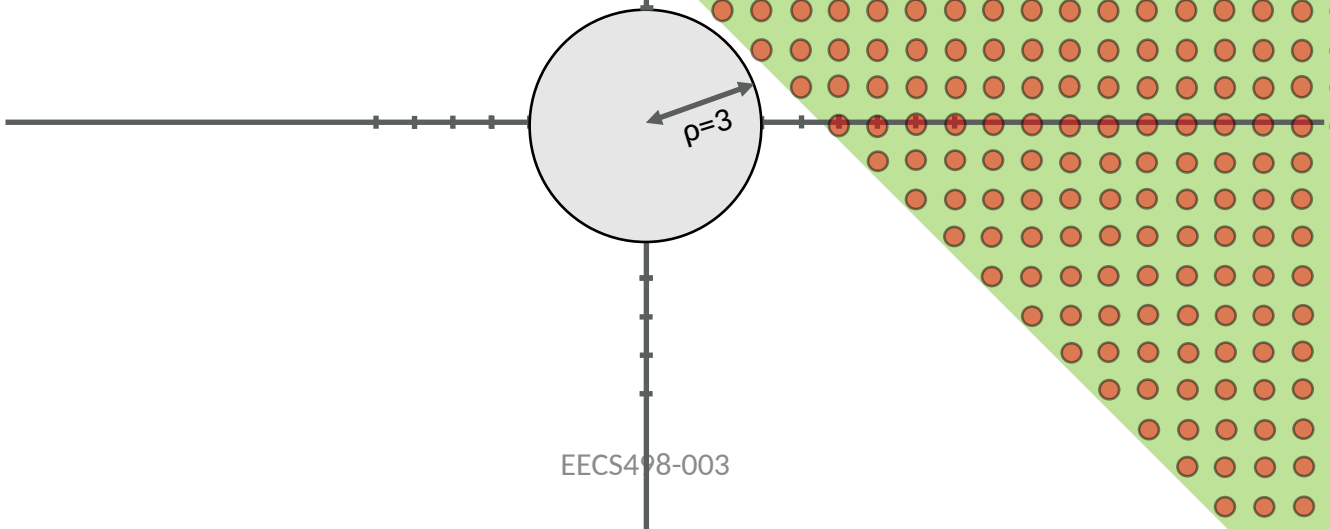


Safe states

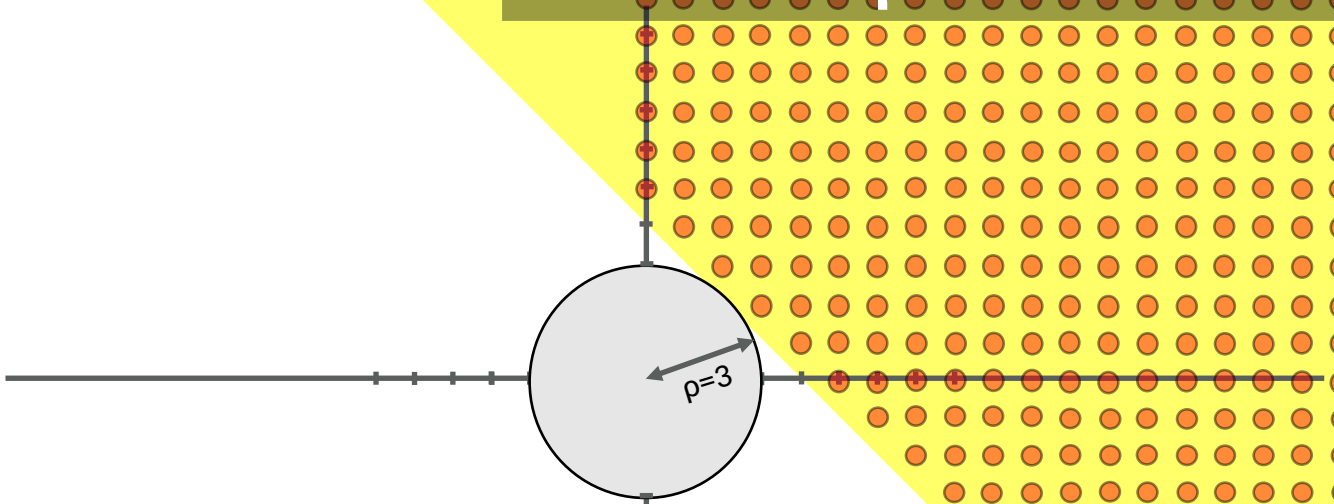
Safety proof using a stronger invariant

InGreenRegion(v) && Next(v, v') ==>
InGreenRegion(v')

The set of reachable states is always an inductive invariant

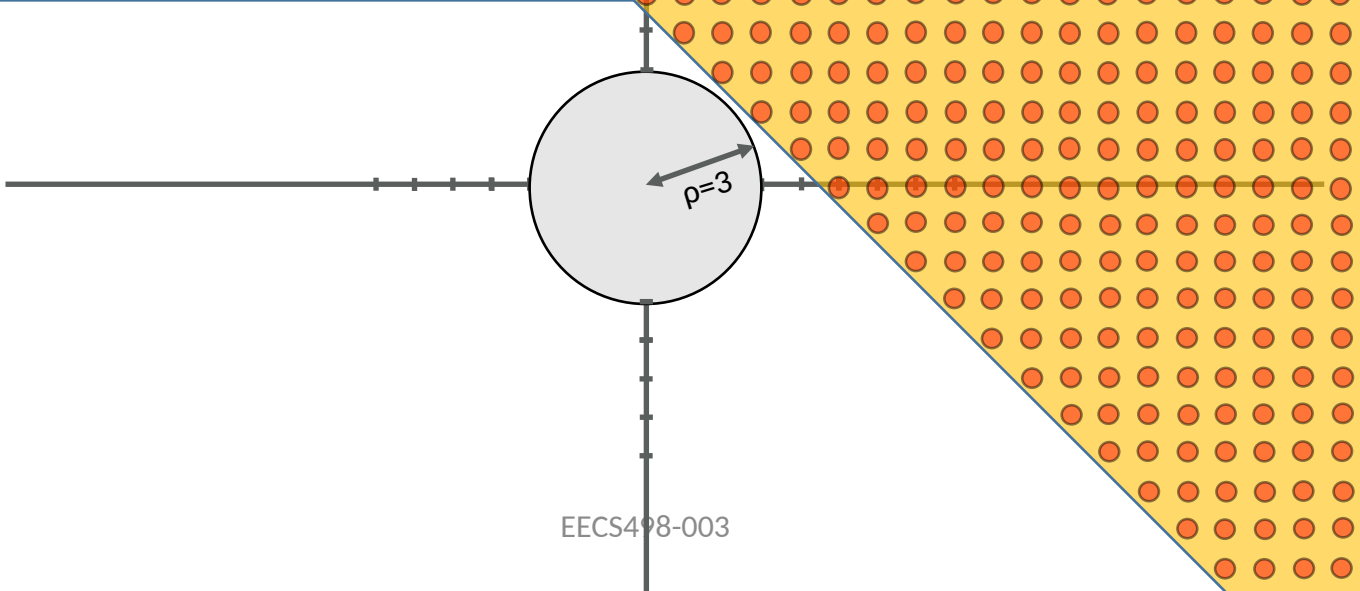


A simpler inductive invariant

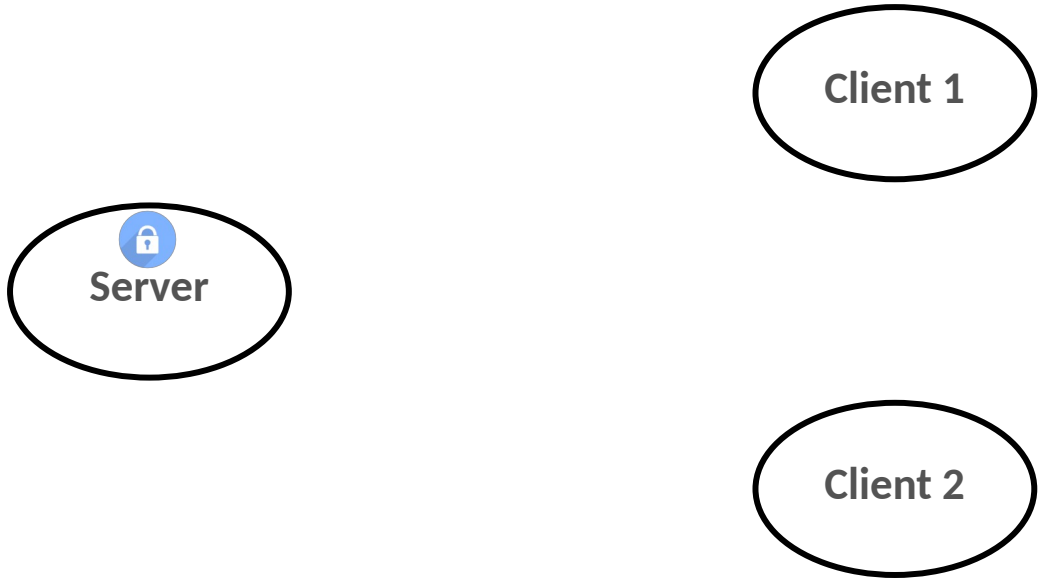


InYellowRegion(v) && Next(v, v') ==>
InYellowRegion(v')

A candidate inductive invariant



Example: lock server

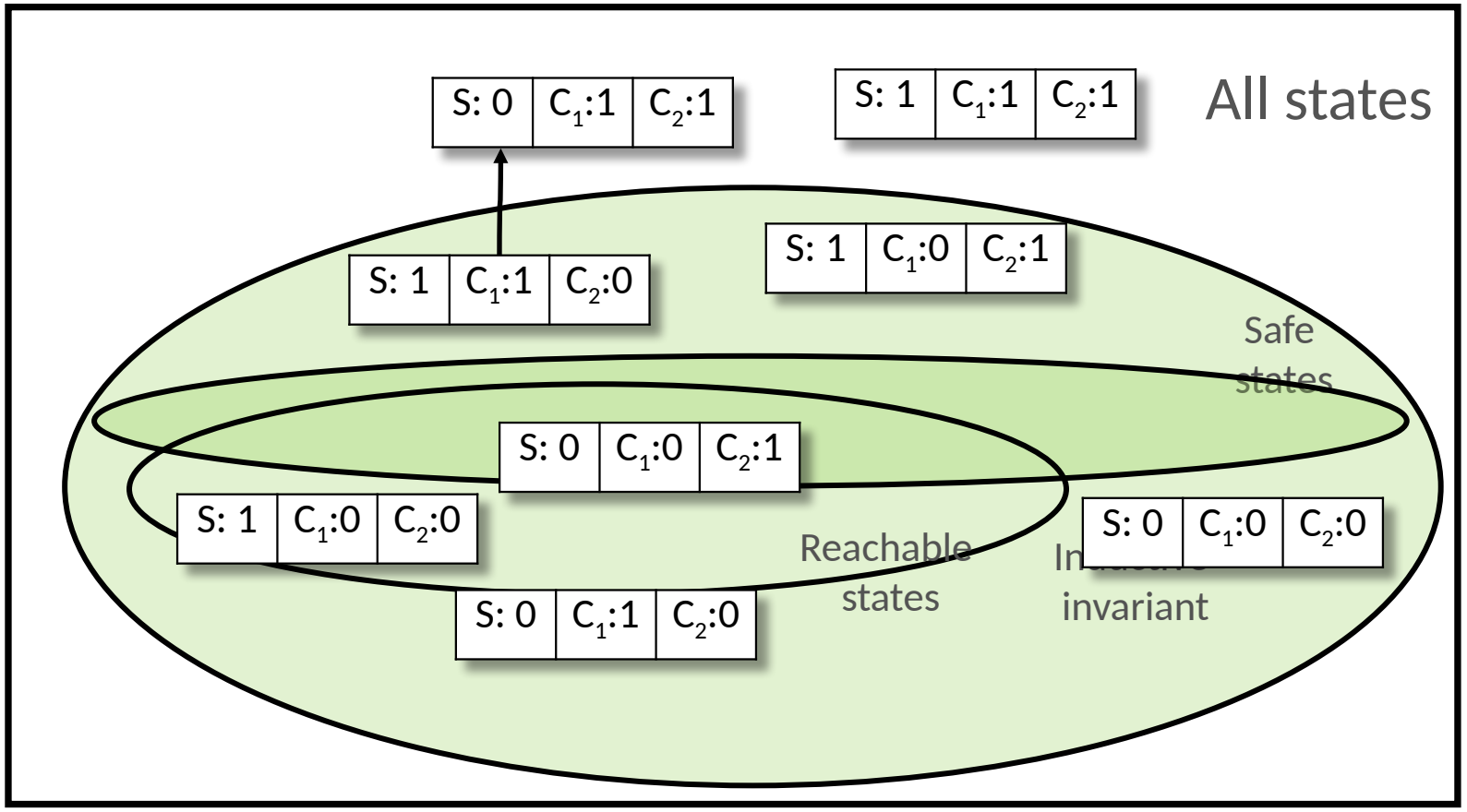


datatype Variables = Variables(S: bool, C1: bool, C2:bool)

ghost predicate Safety(v) { !(v.C1 && v.C2) }

Both clients cannot hold the lock at the same time

Example: lock server



Administrivia

- Chapter 4 will be released today
  - PS2 is due October 3

- Keep in mind that each chapter is a separate submission on the AG
  - And thus consumes late days separately

Some useful boilerplate

```
datatype Constants = Constants(capacity:int)
datatype Variables = Variables(numCokes:int)

predicate Init(c:Constants, v:Variables) { ... }

predicate Next(c:Constants, v:Variables, v':Variables) { ... }
```

Some useful boilerplate

```
datatype Constants = Constants(tableSize:nat)
{
  predicate WellFormed() {
      && 0 < tableSize
  }
}
```

```
datatype Variables = Variables()
{
  predicate WellFormed(c:
Constants) {
    && c.WellFormed()
  }
}
```

Typical examples:

- Length constraints on sequences
- Indices fit into a sequence length
- Domains of maps

Non-linear arithmetic

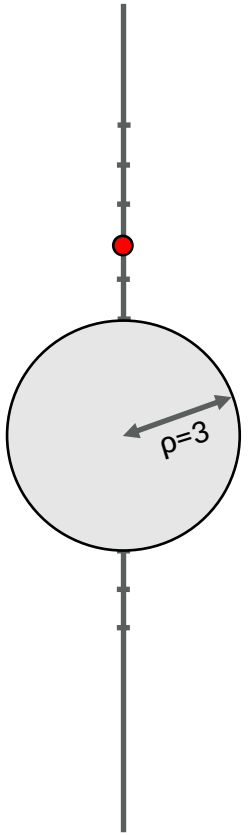
Dafny runs without non-linear reasoning by default

Beware of modulo operations

- Think of alternatives, if you run into trouble

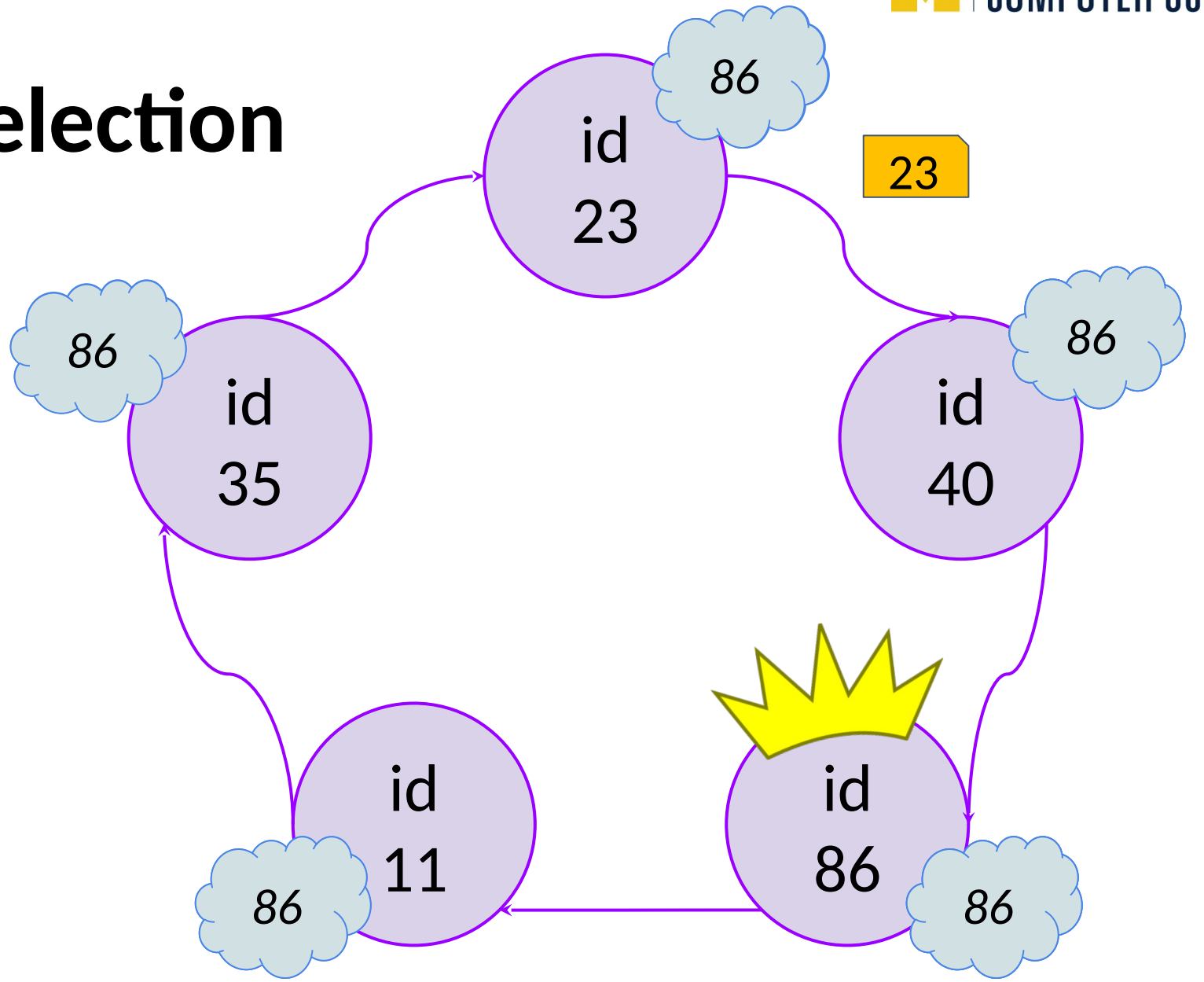
Crawler 2: Revenge of the inductive invariant

- The crawler can now only move North/South
  - Initially it can only move North
- It can also Flip(), teleporting to the symmetric point on the y-axis and changing direction of movement

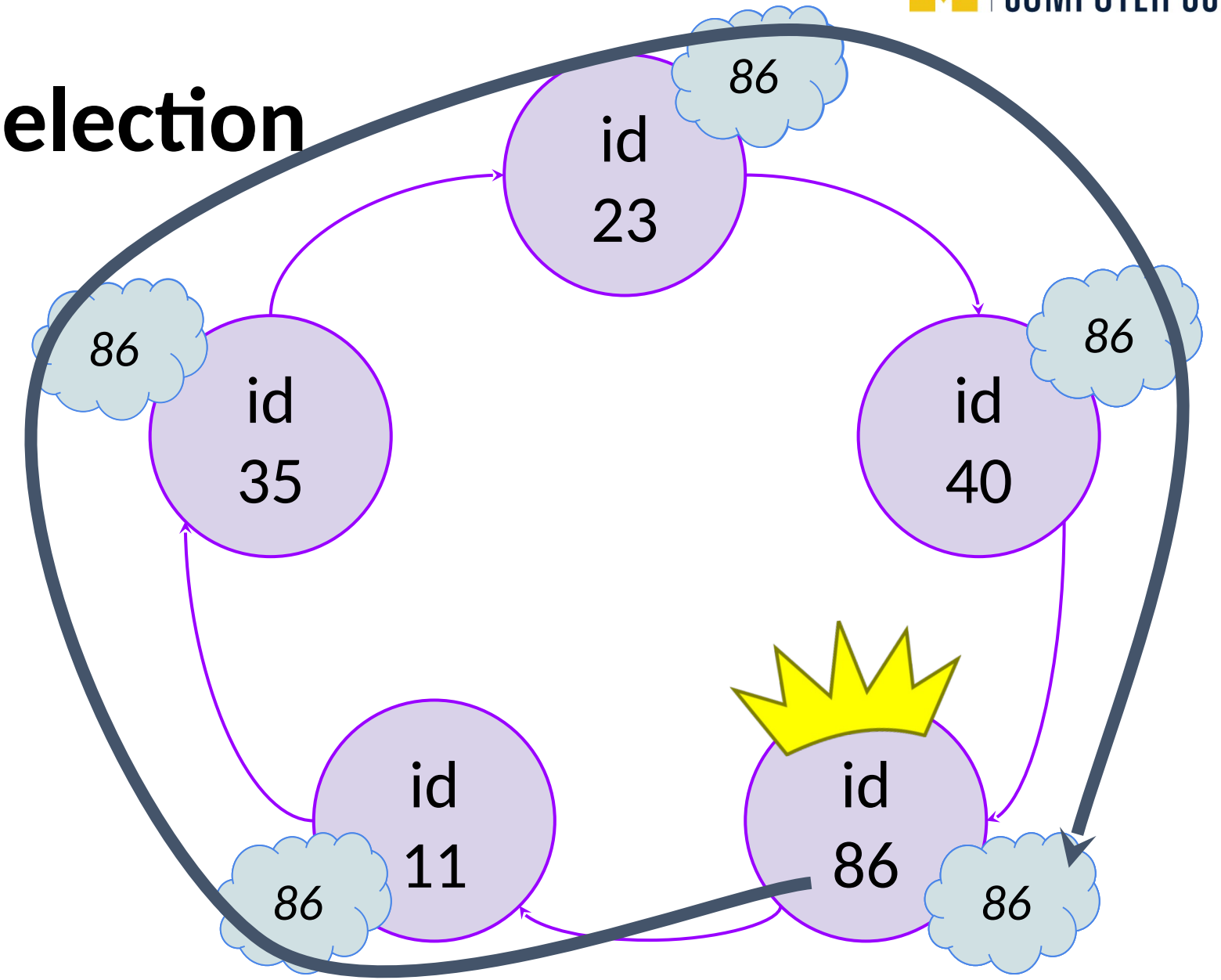


VSCode transition

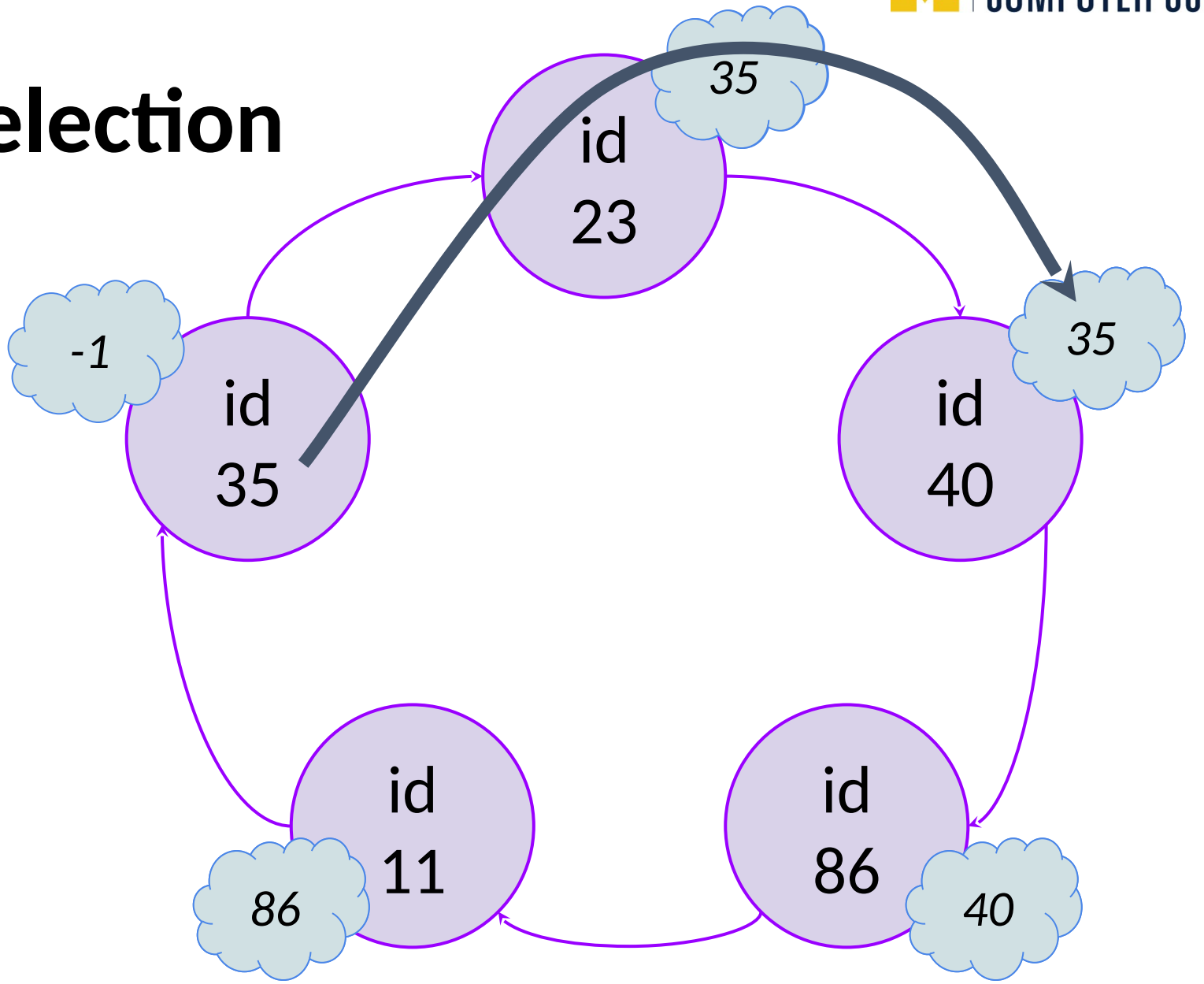
Leader election



Leader election



Leader election



Leader election