EECS498-003
Formal Verification of Systems Software

Material and slides created by

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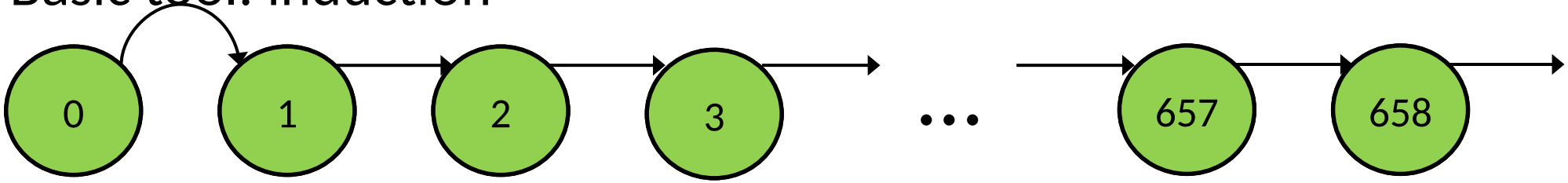
Chapter 4: Proving properties

Expressing a system as a state machine allows us to prove that it has certain properties

- We will focus on safety properties
  - i.e. properties that hold throughout the execution

Basic tool: induction



- Show that the property holds on state 0
- Show that if the property holds on state k, it must hold on state k+1

Proving a safety invariant

```
predicate Safety(v:Variables) {
  true // TBD
}

lemma SafetyProof()
  ensures forall v :: Init(v) ==> Safety(v)
  ensures forall v, v' :: Safety(v) && Next(v, v') ==> Safety(v')
{
}
```

Base case

VSCode transition

Inductive Step

Jay Normal Form

As you begin writing more interesting specs, proofs will be nontrivial.

Pull all the nondeterminism into one place, and get a receipt.

image: flickr/afagen CC-by-nc-sa

Jay Normal Form

```
datatype Step =
  | Action1Step( <parameters> )
  | Action2Step( <parameters> )
  ...

predicate NextStep(v: Variables, v': Variables, step:Step)
{
  match step
    case Action1Step(<parameters>) => Action1(v, v', <parameters>)
    case Action2Step(<parameters>) => Action2(v, v', <parameters>)
    ...
}
predicate Next(v: Variables, v': Variables)
{
    exists step :: NextStep(v, v', step)
}
```

VSCode transition

Administrivia

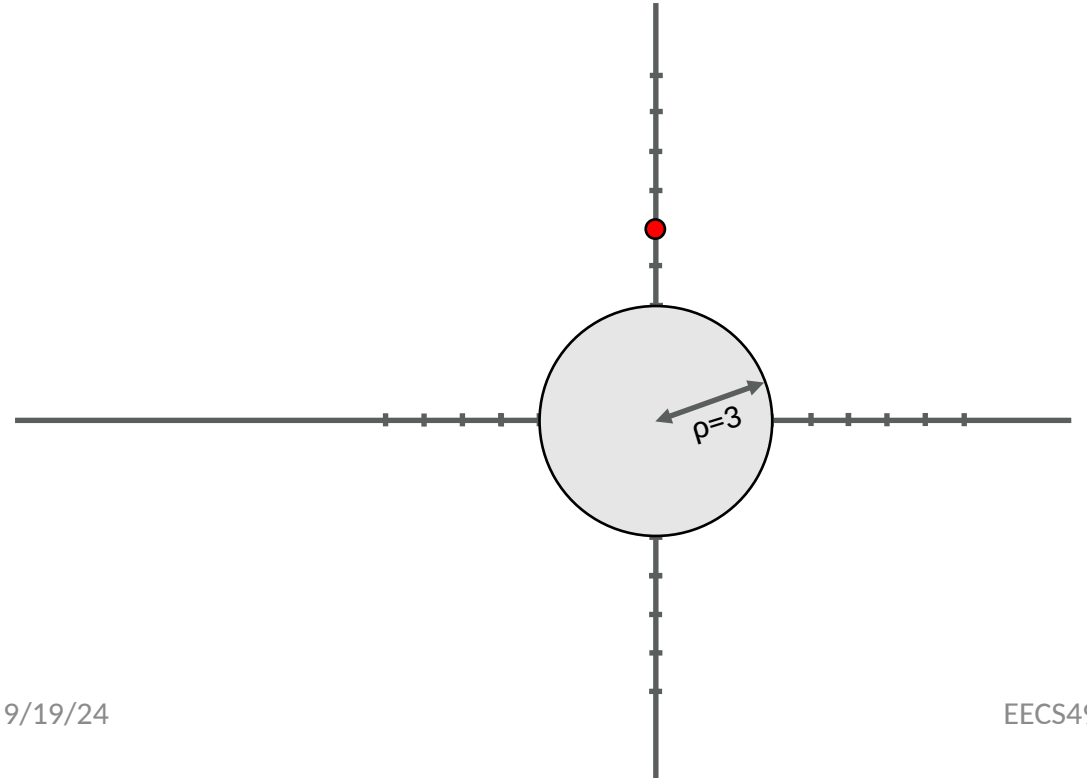
- I have to leave right after class today

- Problem Set 1 due today
- Problem Set 2 will be released tomorrow
  - Chapters 3 and 4
    - Due October 3, 11:59pm
- Reminder: assignment timeline on Piazza

- Not too early to start finding a partner for Project 1

A simple application: Crawler

- Crawler starts at (0,5)
- It can move 1 step north or 1 step south-east
- Can it ever fall in the hole?




```
predicate Init(v:Variables) {
   && v.x == 0
   && v.y == 5
 }

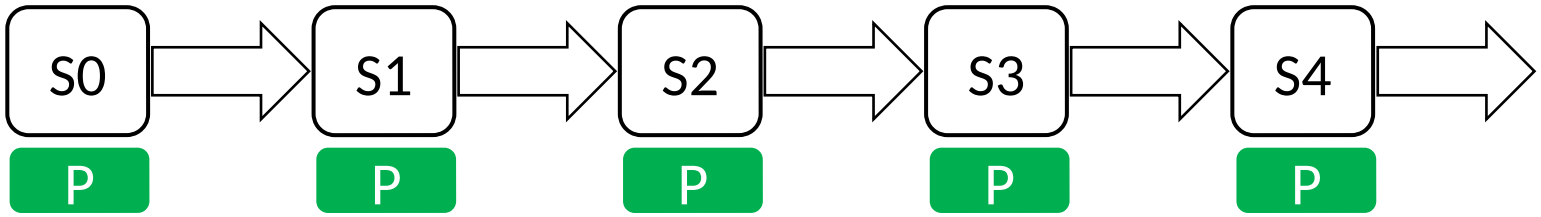
 predicate MoveNorth(v:Variables, v':Variables) {
   && v'.x == v.x
   && v'.y == v.y + 1
 }

 predicate MoveSouthEast(v:Variables, v':Variables) {
   && v'.x == v.x + 1
   && v'.y == v.y - 1
 }
```

Proving invariants

Proof by induction

- Prove it holds on the first state
- Prove it holds during a transition



Init(v) ==> P(v)

P(v) && Next(v, v') ==> P(v')

Proving the Crawler

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predicate Init(v:Variables) {
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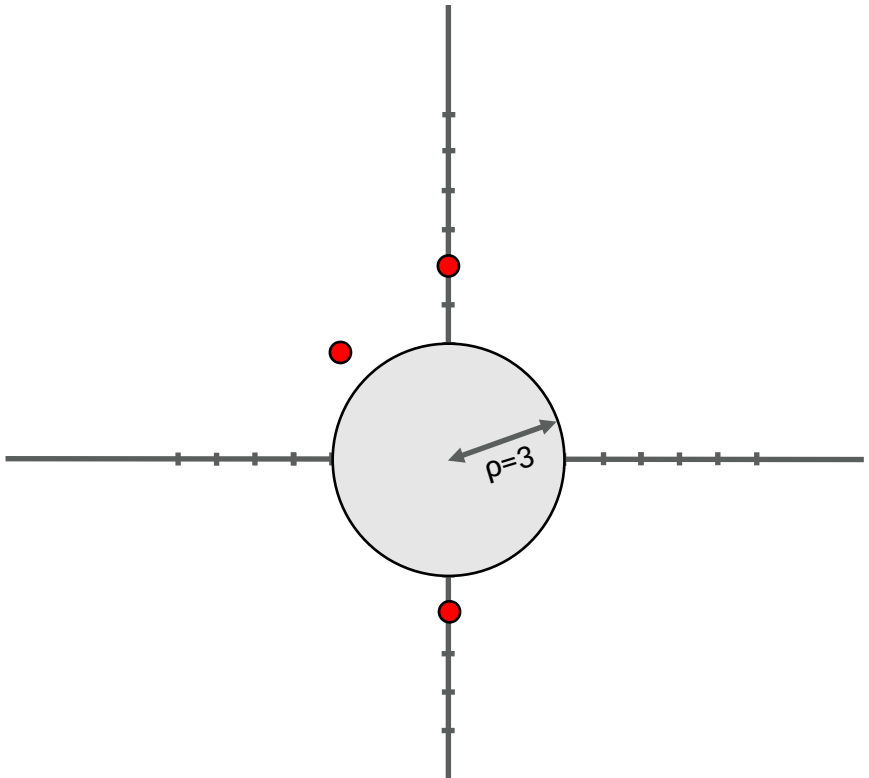
 predicate MoveSouthEast(v:Variables, v':Variables) {
   && v'.x == v.x + 1
   && v'.y == v.y - 1
 }
```

```
predicate Safety(v:Variables) {
    v.x*v.x + v.y*v.y > 3*3
}
```

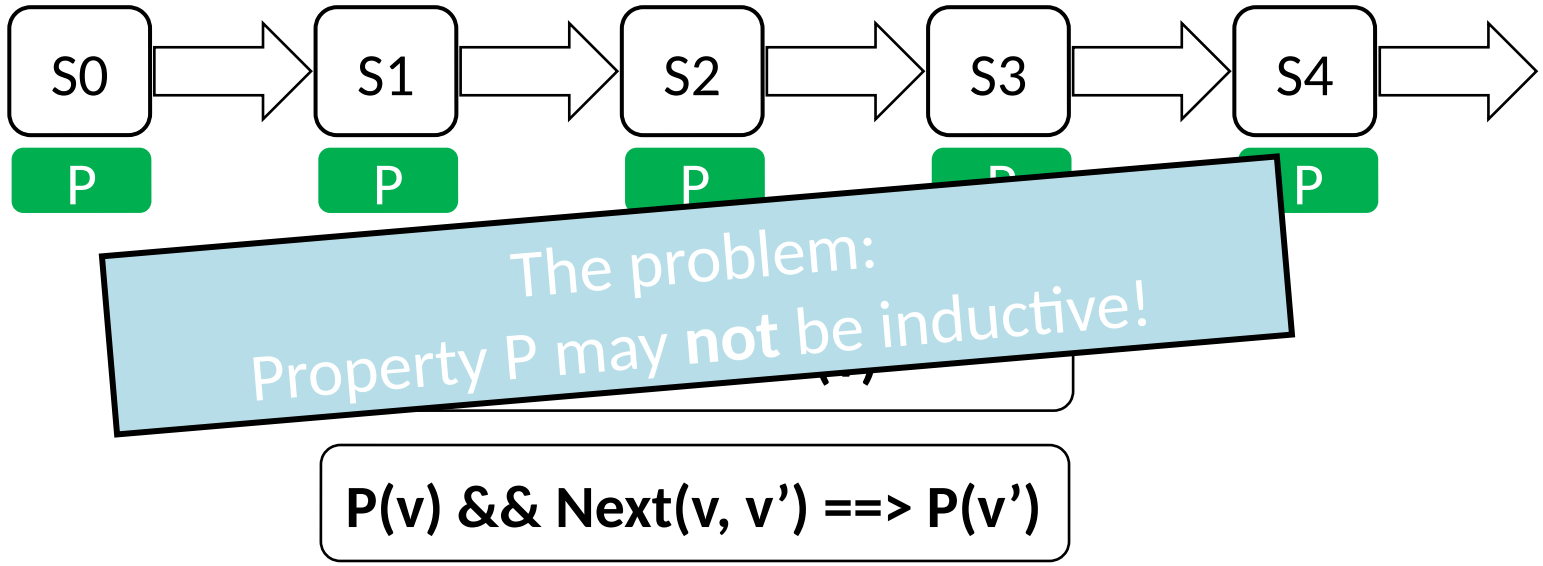
Init(v) ==> P(v) ✓

P(v) && Next(v, v') ==> P(v') ✗

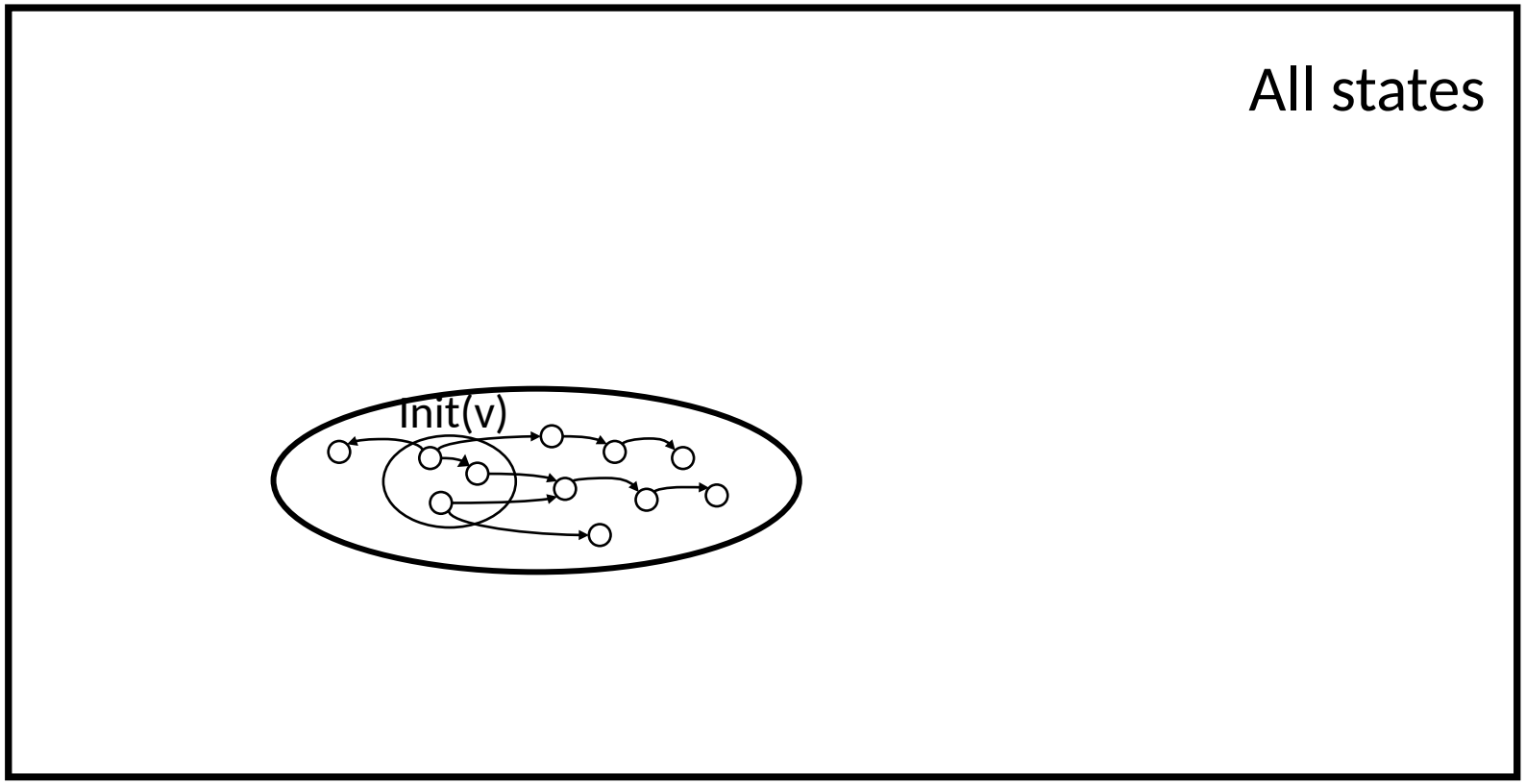


Inductive invariants

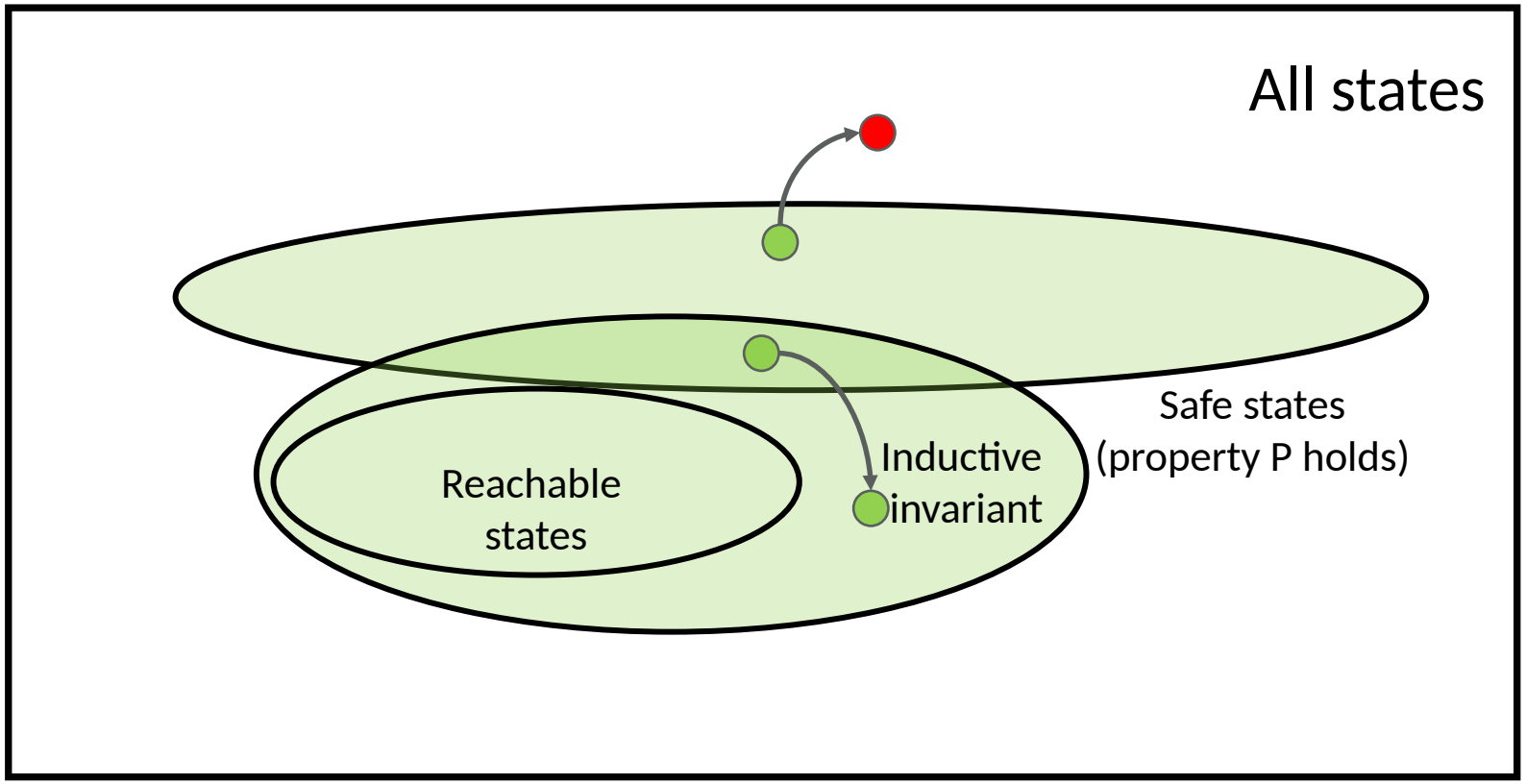
Safety property (a.k.a. invariant):
a property that **always** holds



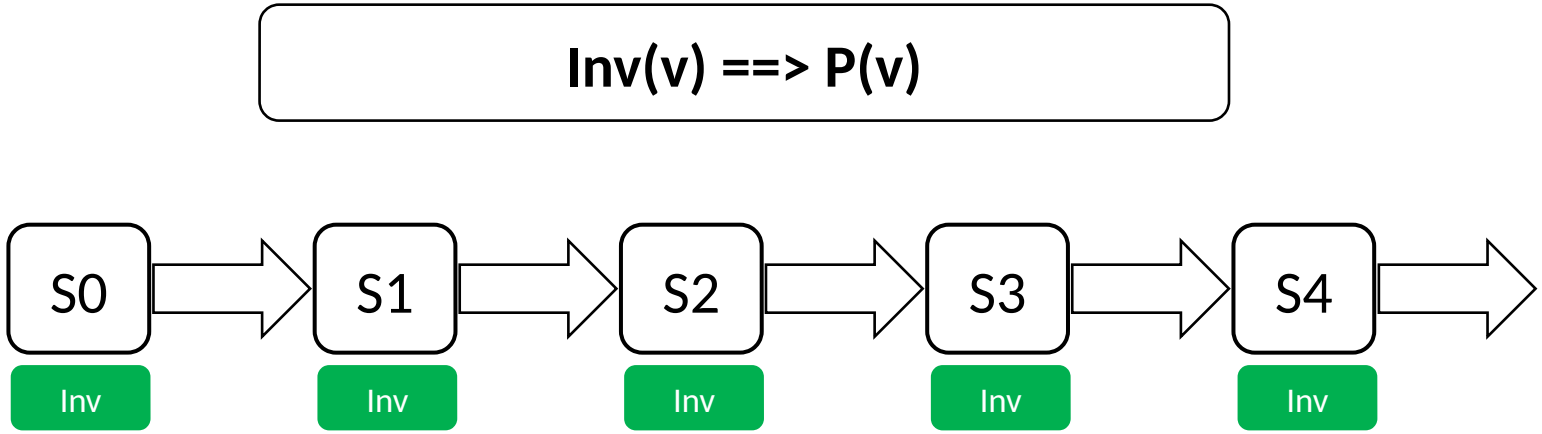
Invariants vs Inductive invariants



Invariants vs Inductive invariants



Proving safety with Inductive invariants

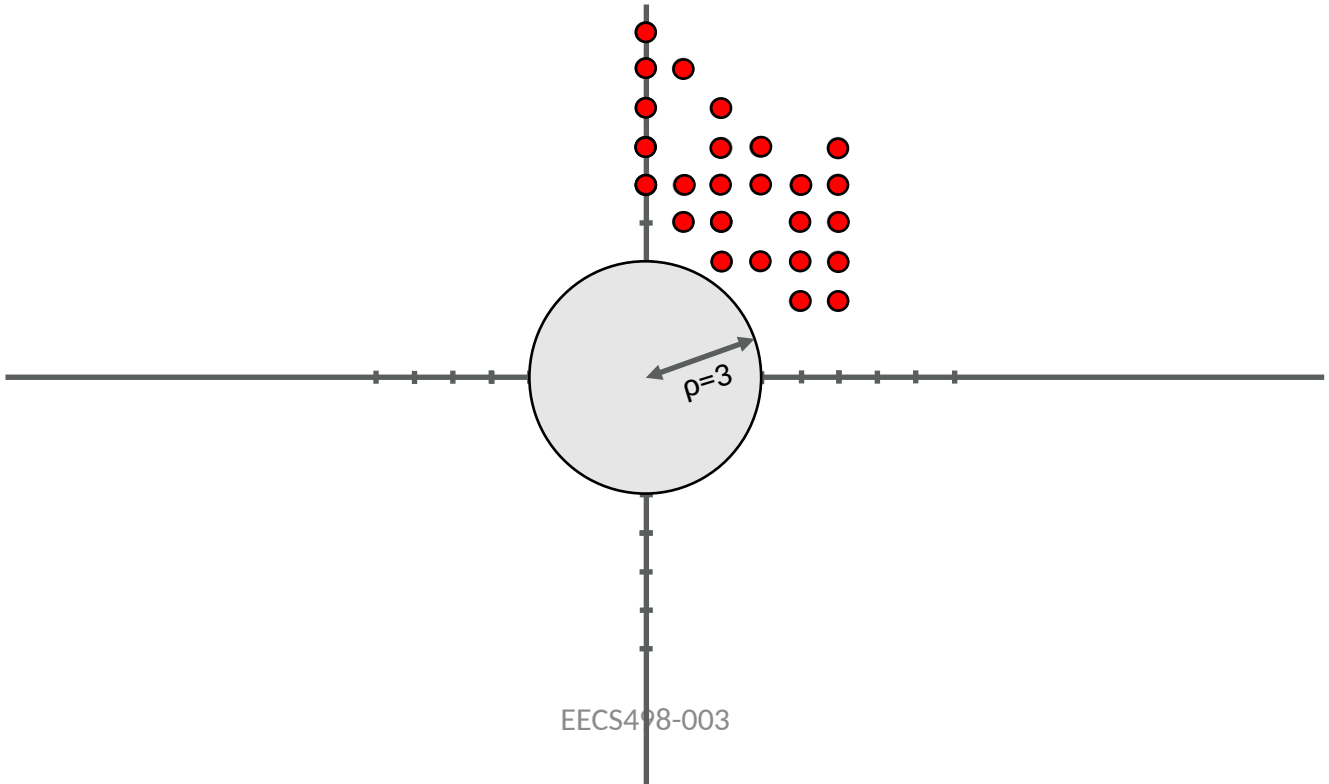


Init(v) ==> Inv(v)

Inv(v) && Next(v, v') ==> Inv(v')

Proving the Crawler

Can the crawler ever fall in the hole?



Proving the Crawler

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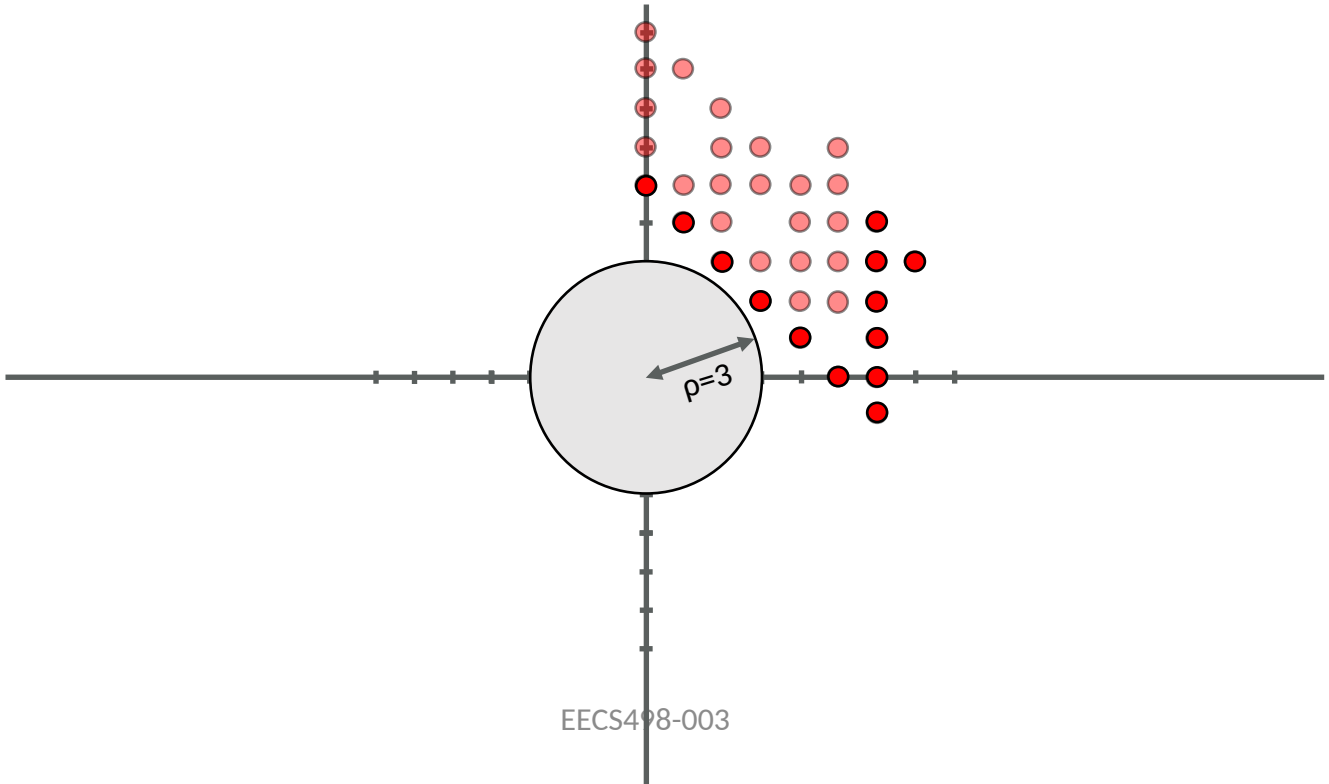

Proving the Crawler

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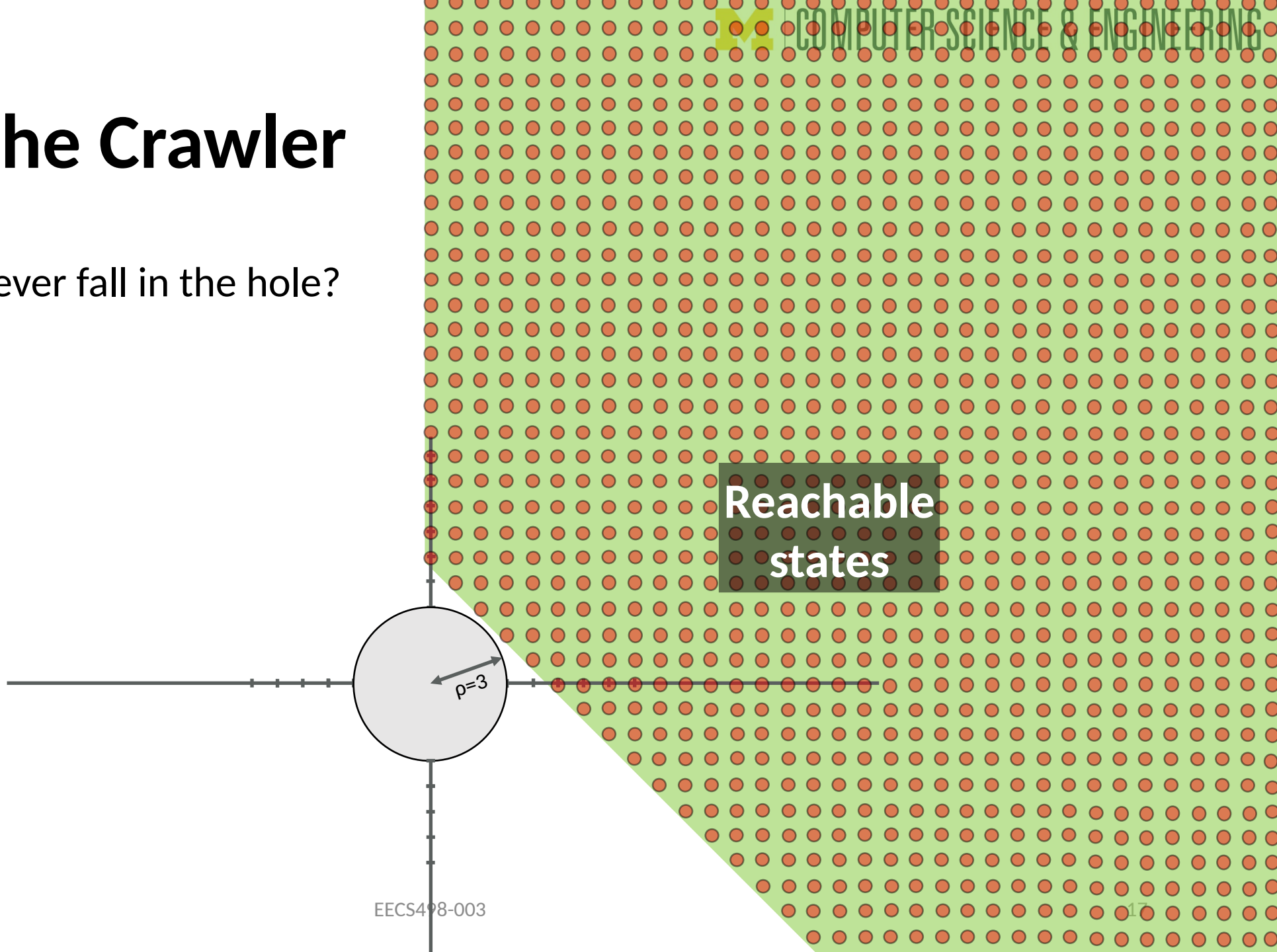
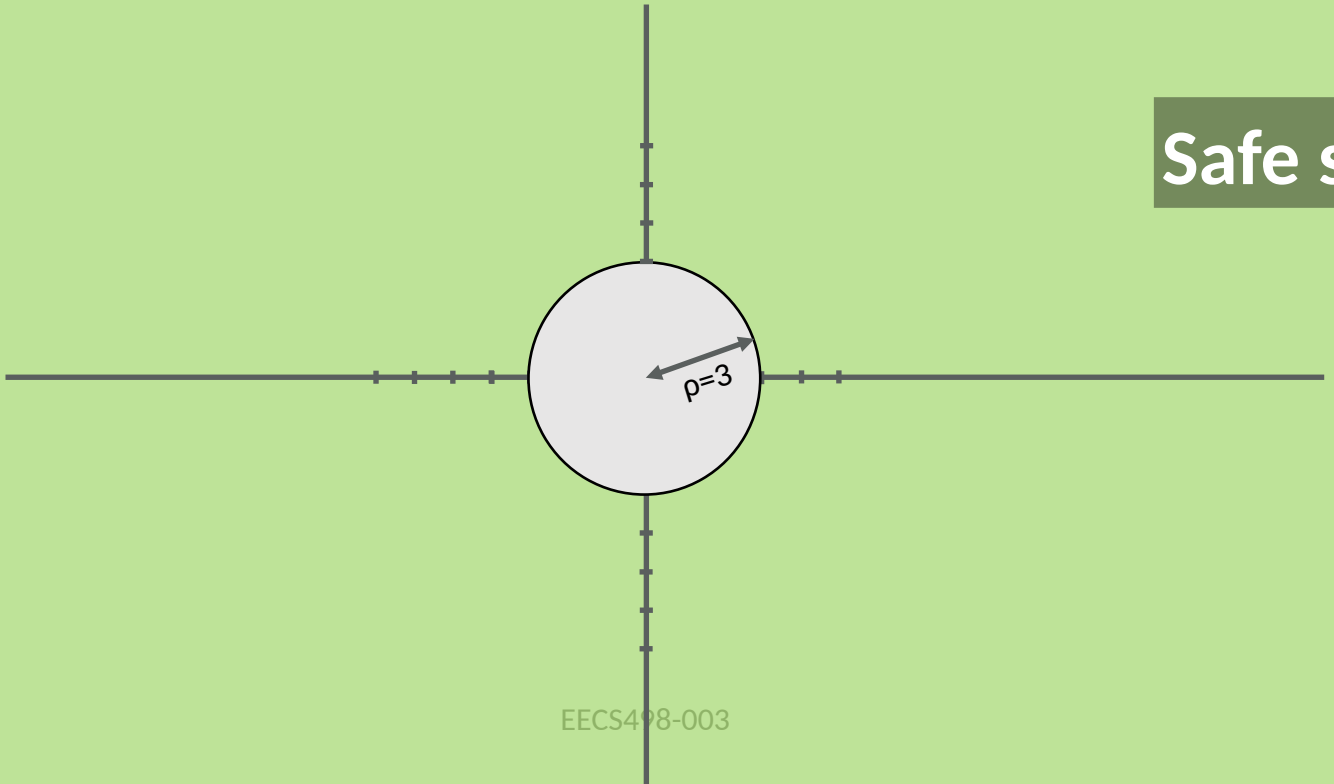


Naïve safety proof

Safe(v) && Next(v, v') ==> Safe(v')

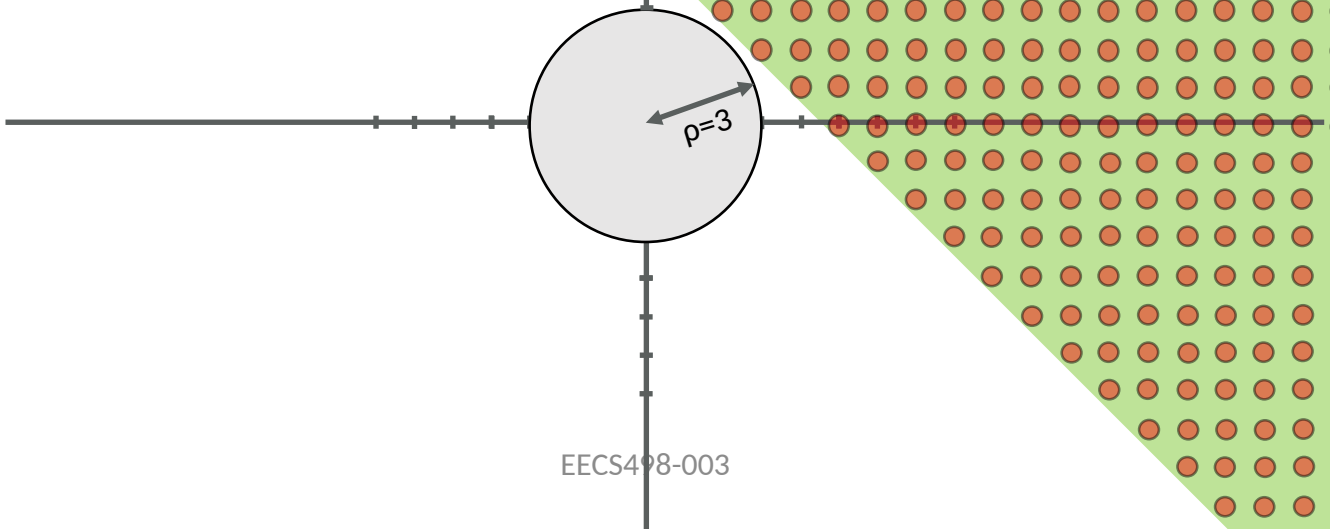


Safe states

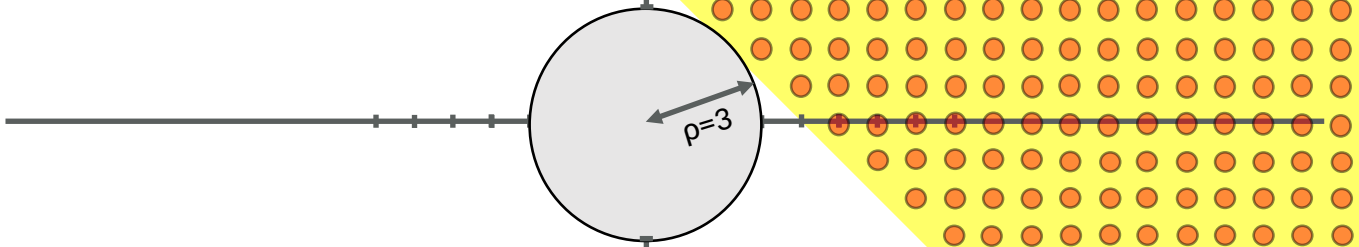
Safety proof using a stronger invariant

InGreenRegion(v) && Next(v, v') ==>
InGreenRegion(v')

The set of reachable states is always an inductive invariant



A simpler inductive invariant



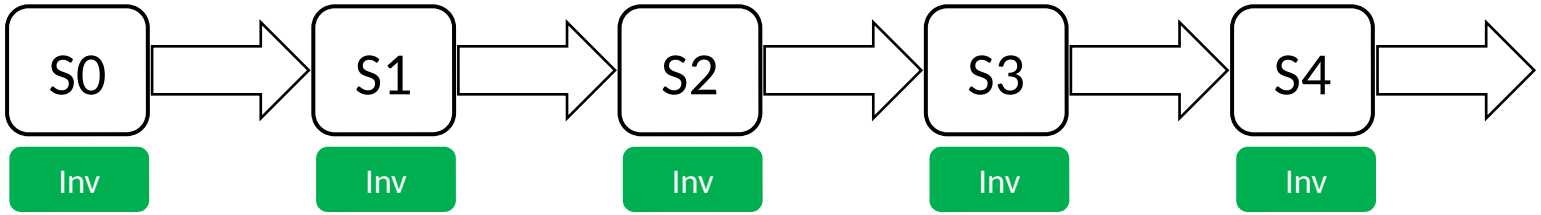
InYellowRegion(v) && Next(v, v') ==>
InYellowRegion(v')

Proving safety with Inductive invariants

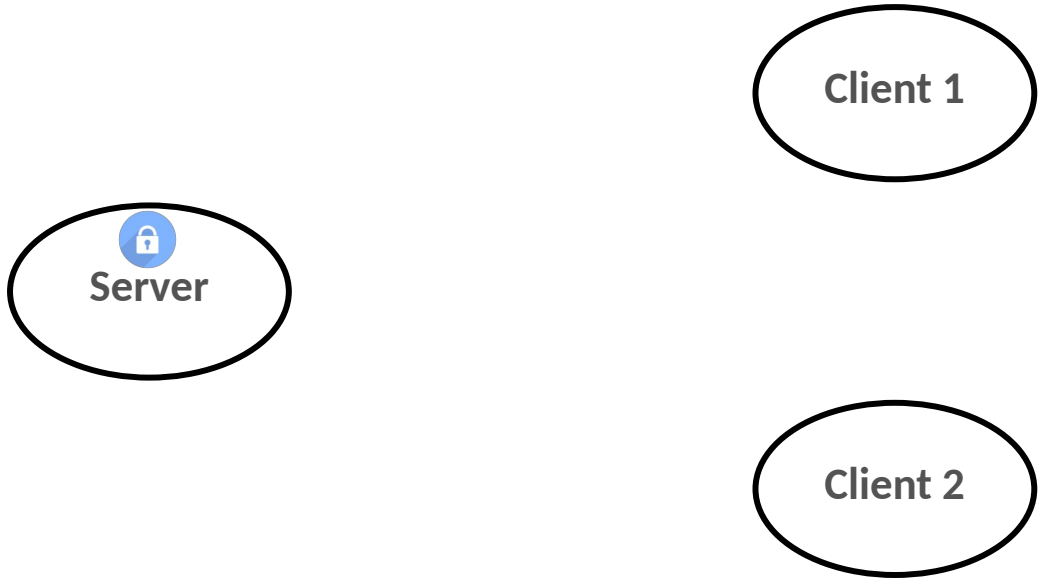
Inv(v) ==> Safety(v)

Init(v) ==> Inv(v)

Inv(v) && Next(v, v') ==> Inv(v')



Example: lock server



datatype Variables = Variables(S: bool, C1: bool, C2:bool)

ghost predicate Safety(v) { !(v.C1 && v.C2) }

Both clients cannot hold the lock at the same time

Example: lock server