

# **EECS498-008**

# **Formal Verification**

# **of Systems Software**

Material and slides created by  
Jon Howell and Manos Kapritsos

# Some new Dafny syntax

## Datatype member functions

```
datatype Pet = Dog | Cat | Ant | Spider {  
  function CountLegs() : int {  
    match this  
      case Dog => 4  
      case Cat => 4  
      case Ant => 6  
      case Spider => 8  
    }  
  }  
}  
  
function ShoesForTwo(pet: Pet) : int {  
  2 * pet.CountLegs()  
}
```

# Some new Dafny syntax

## Calc statements

```
assert a == b;  
assert b == c;  
assert c == d;
```

```
calc {  
  a;  
  b;  
  c;  
  d;  
}
```

# Some new Dafny syntax

## Calc statements

```
assert a == b;  
assert b == c;  
assert c == d;
```

```
calc {  
  a;  
  { MyUsefulLemma(a,b); }  
  b;  
  c;  
  d;  
}
```

# Some new Dafny syntax

## Calc statements

```
assert a == b;  
assert b == c;  
assert c == d;
```

```
calc ==> {  
  a;  
  { MyUsefulLemma(a, b); }  
  b;  
  c;  
  d;  
}
```

# Some new Dafny syntax

Choose operator

```
assert 1 % 7 == 1;  
assert exists x :: x % 7 == 1;  
var x :| x % 7 == 1;
```

Choose x such that...

# Administrivia

Remember that Problem Set 1 is due this Friday

I'm still missing some of your pictures. Please send me your picture ([manosk@umich.edu](mailto:manosk@umich.edu)) with the Subject “EECS498-008 picture”

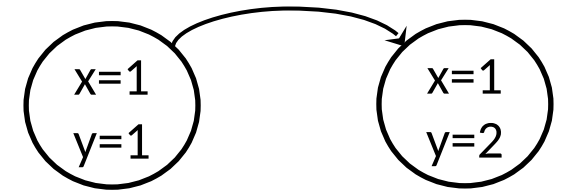
# Chapter 3: Building state machines

A **state** is an assignment of values to variables

An **action** is a transition from one state to another

An **execution** is a sequence of states

We will capture executions with **state machines**





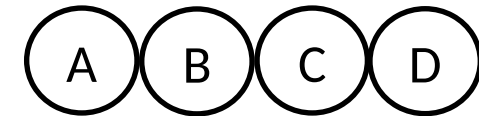
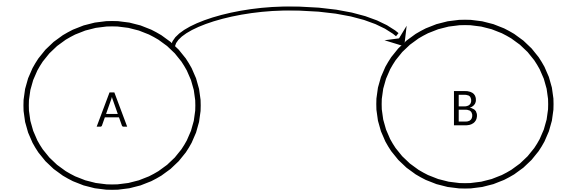
# Building state machines

A **state** is an assignment of values to variables

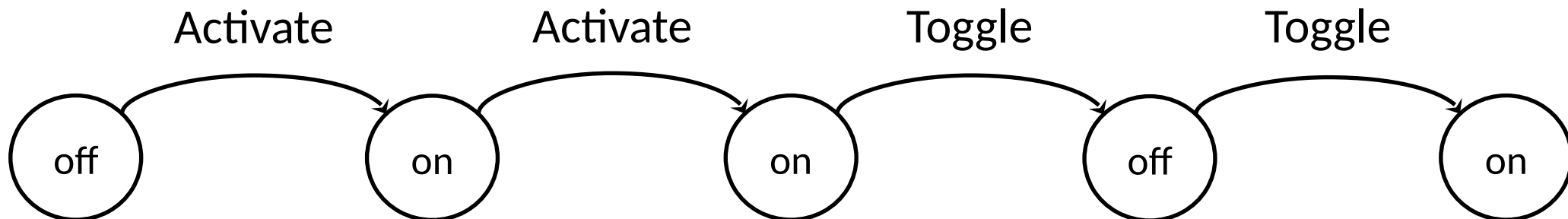
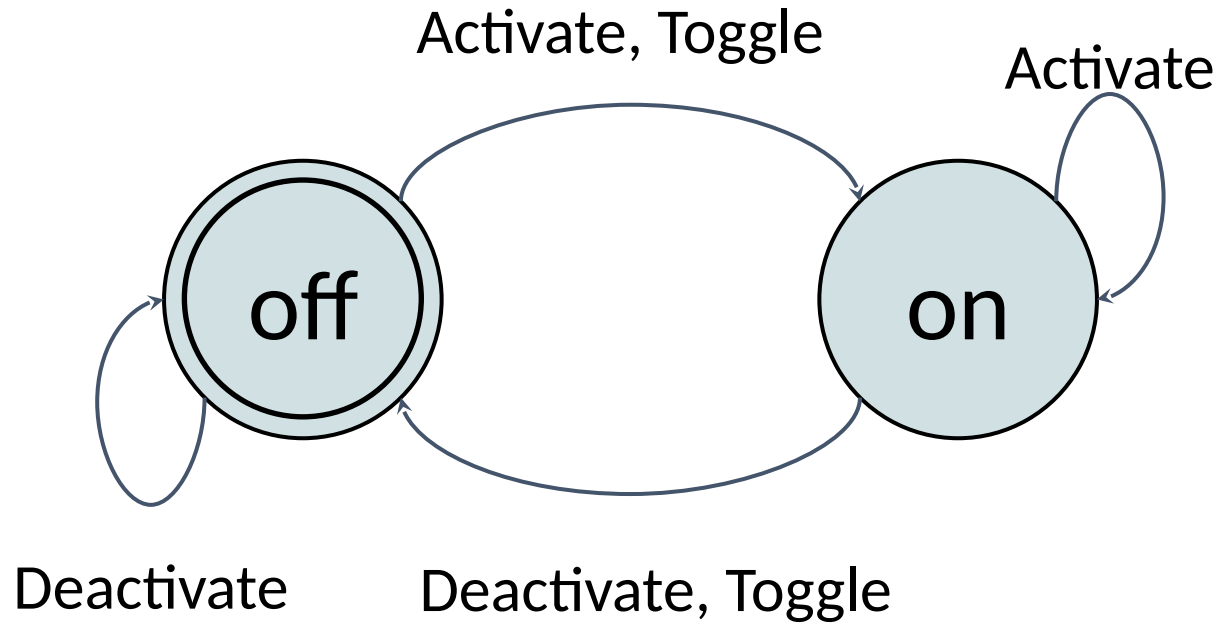
An **action** is a transition from one state to another

An **execution** is a sequence of states

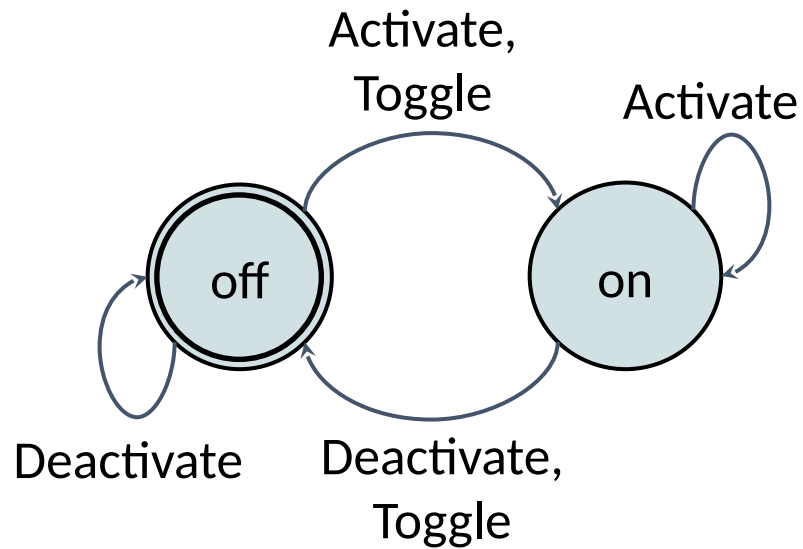
We will capture executions with **state machines**



# The Switch state machine



# The Switch state machine



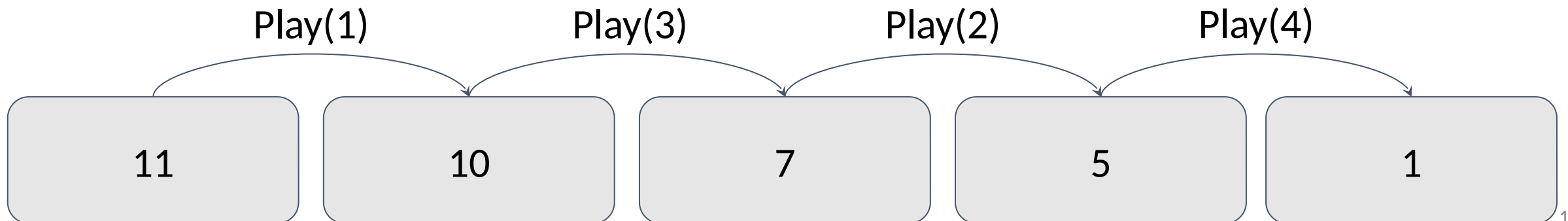
```

datatype SwitchState = On | Off
datatype Variables =
    Variables(switch:SwitchState)
predicate Init(v:Variables) {
    v.switch == Off
}
  
```

```

predicate Activate(v:Variables, v':Variables) {
    v'.switch == On
}
predicate Deactivate(v:Variables, v':Variables) {
    v'.switch == Off
}
predicate Toggle(v:Variables, v':Variables) {
    On v'.switch == if v.switch.On? then Off else
    On
}
predicate Next(v:Variables, v':Variables) {
    || Activate(v, v')
    || Deactivate(v, v')
    || Toggle(v, v')
}
  
```

# The Game of Nim



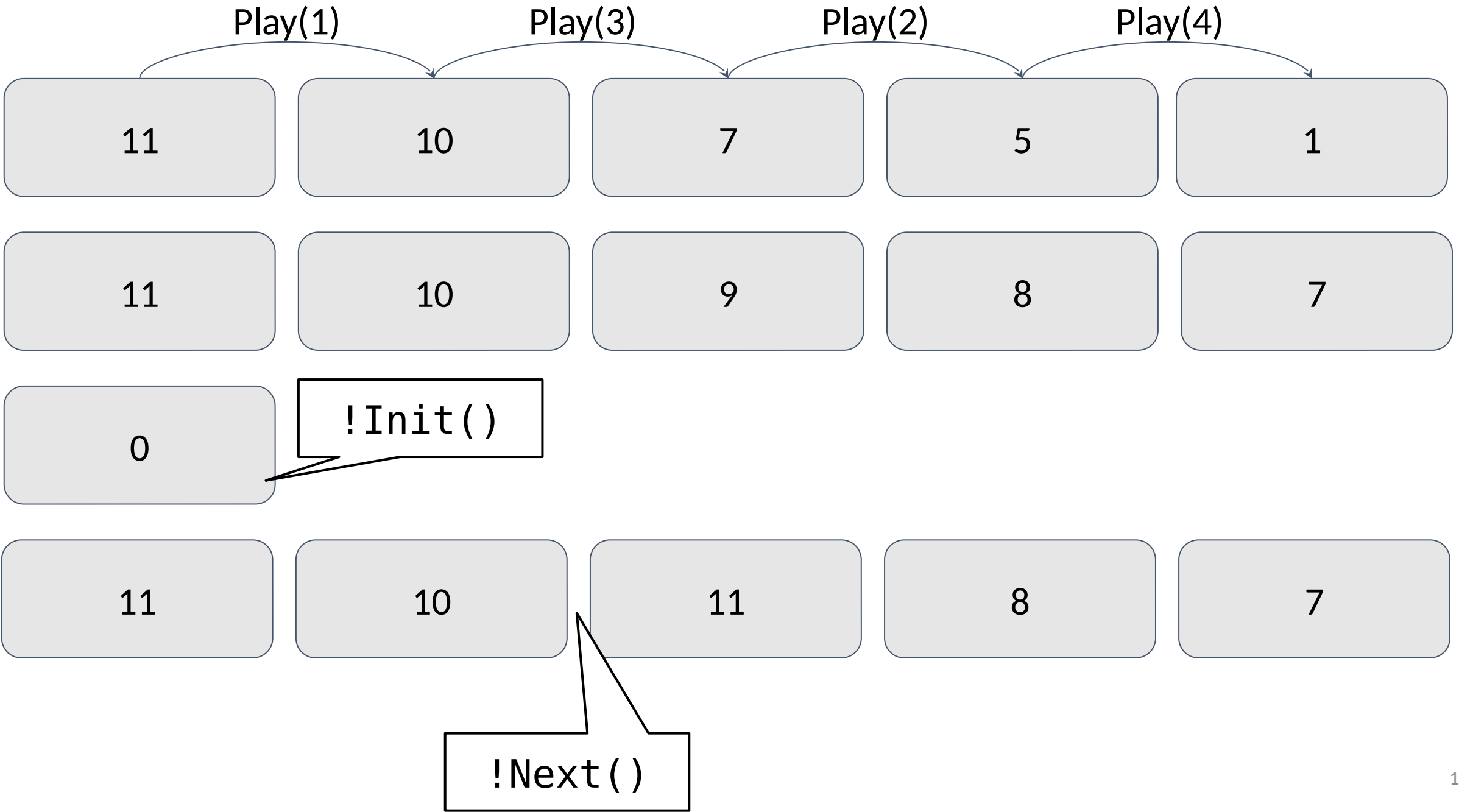
# The Nim state machine

```
datatype Variables = Variables(tokens:nat)
predicate Init(v:Variables) {
  v.tokens > 0
}
```

```
predicate Play(v:Variables, v':Variables, take:nat) {
  && 1 <= take <= 4
  && take <= v.tokens
  && v'.tokens == v.tokens - take
}
```

} enabling condition  
"update"

```
predicate Next(v:Variables, v':Variables)
{
  exists take :: Play(v, v', take)
}
```



# A **state** is an assignment of values to variables

```
datatype Card = Shelf | Patron(name: string)
datatype Book = Book(title: string)
type Library = map<Book, Card>
```

The **state space** is the set of possible assignments.

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Jon

~~The Martian: Jon  
Snow Crash: Jon~~

The Martian: Manos  
Snow Crash: Jon

# An **execution** is an infinite sequence of states

check out

check out

check in

check out

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Jon

The Martian: Manos  
Snow Crash: Jon

The Martian: Shelf  
Snow Crash: Jon

The Martian: Rob  
Snow Crash: Jon

check out

check in

check out

check in

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Jon  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Rob  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Shelf

check out

???

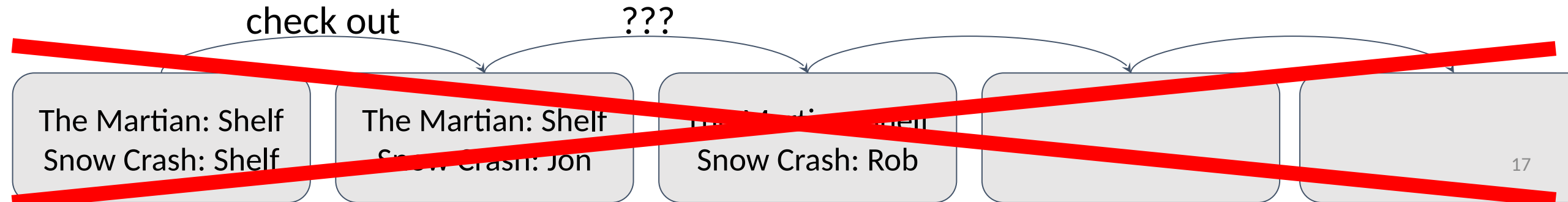
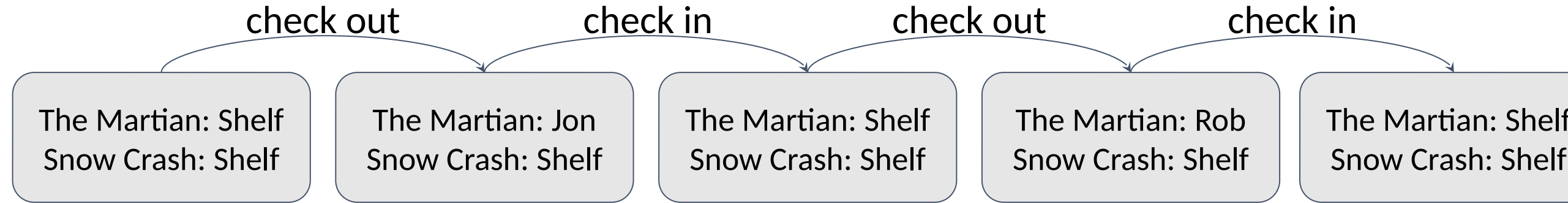
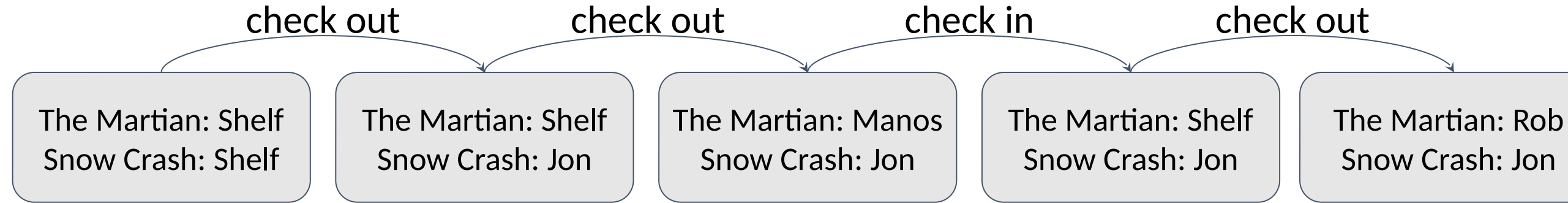
The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Jon

The Martian: Shelf  
Snow Crash: Rob



# A behavior is the set of all possible executions



# How should we define a behavior?

With a **program**?

Its variables define its state space  
Its executions define its behavior

Weaknesses:

- concreteness
- nondeterminism
- asynchrony
- environment

# How should we define a behavior?

With a state machine

Its **type** defines its state space

Its **initial states** and **transitions** define its behavior

# A state machine definition

```
datatype Card = Shelf | Patron(name:
string)
datatype Book = Book(title: string)
type Library = map<Book, Card>
```

```
predicate Init(v: Library) {
  forall book | book in v :: v[book] == Shelf
}
predicate CheckOut(v : Library, v' : Library, book: Book, name: string) {
  && book in v
  && v[book] == Shelf
  && (forall book | book in v :: v[book] != Patron(name))
  && v' == v[book := Patron(name)]
}
predicate CheckIn(v : Library, v' : Library, book: Book, name: string) {
  && book in v
  && v[book] == Patron(name)
  && v' == v[book := Shelf]
}
predicate Next(v: Library, v': Library) {
  || (exists book, name :: CheckOut(v, v', book, name))
  || (exists book, name :: CheckIn(v, v', book, name))
}
```

} enabling condition  
} "update"  
} Nondeterministic definition

# A behavior is the set of all possible executions

```

predicate CheckOut(v, v', book, name) {
  && book in v
  && v[book] == Shelf
  && (forall book | book in v :: v[book] !=
Patron(name))
  && v' == v[book := Patron(name)]
}
predicate CheckIn(v, v', book, name) {
  && book in v
  && v[book] == Patron(name)
  && v' == v[book := Shelf]
}

```

check out

???

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Jon

The Martian: Shelf  
Snow Crash: Rob

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Shelf

The Martian: Shelf  
Snow Crash: Shelf

# State machine strengths

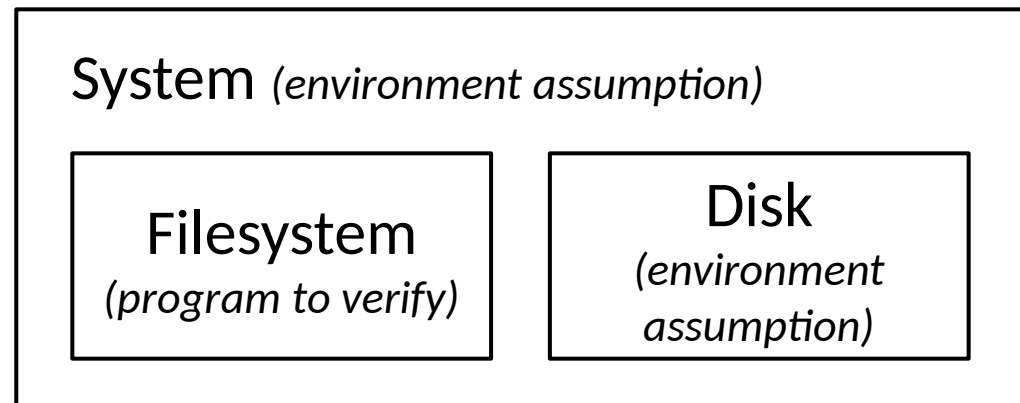
- Abstraction
  - States can be abstract
    - Model an infinite map instead of an efficient pivot table
  - Next predicate is nondeterministic:
    - Implementation may only select some of the choices
    - Can model Murphy's law (e.g. crash tolerance) or an adversary

# State machine strengths

- Abstraction
- Asynchrony
  - Each step of a state machine is conceptually atomic
  - Interleaved steps capture asynchrony: threads, host processes, adversaries
  - Designer decides how precisely to model interleaving; can refine/reduce

# State machine strengths

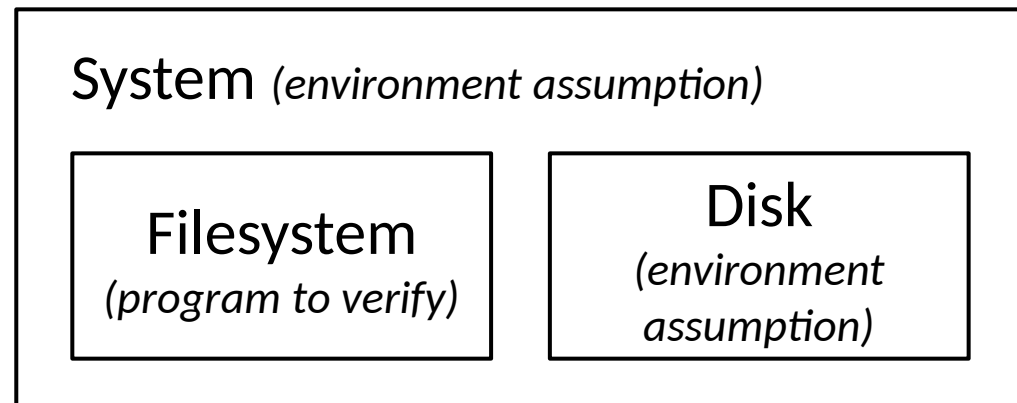
- Abstraction
- Asynchrony
- Environment
  - Model a proposed program with one state machine (verified)
  - Model adversarial environment with another (trusted)
  - Compound state machine models their interactions (trusted)





# State machine strengths

- Abstraction
- Asynchrony
- Environment
  - Model a proposed program with one state machine (verified)
  - Model adversarial environment with another (trusted)
  - Compound state machine models their interactions (trusted)

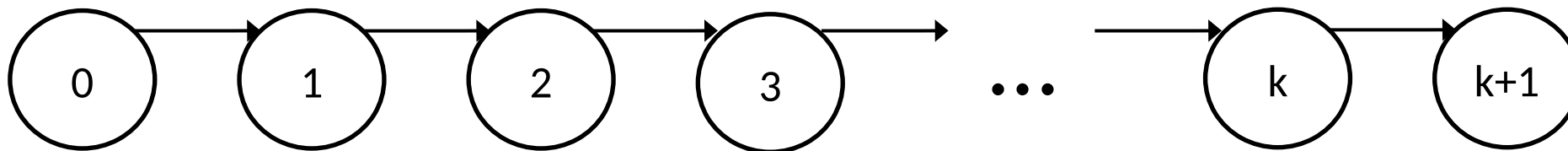


# Chapter 4: Proving properties

Expressing a system as a state machine allows us to **prove** that it has certain properties

- We will focus on safety properties

Basic tool: induction



- Show that the property holds on state 0
- Show that if the property holds on state  $k$ , it must hold on state  $k+1$

# Let's prove a safety invariant!

```
predicate Safety(v:Library) {  
  true // TBD  
}
```

Base case

```
lemma SafetyProof()  
  ensures forall v :: Init(v) ==> Safety(v)  
  ensures forall v, v' :: Safety(v) && Next(v, v') ==> Safety(v')  
{  
}
```

Inductive Step

# Let's prove a safety invariant!

*Interactive proof development in editor*

*Bisection debugging,*

*case analysis,*

*existential instantiation*